No problems labelled H, I, or J.

Problem K. At least once in your mathematical career you should derive the Laplacian in polar and spherical coordinates. If you haven't done so before, do at least the polar form. (The most direct approach is to use the chain rule to get u_x in terms of u_r, r_x, u_θ, and θ_x; then differentiate again with respect to x to get u_{xx}, using the chain and product rules; and similarly with x replaced by y to get u_{yy}. If you use this approach, let me know if you want me to check it, or want to look at my solution. Appendix 4 gives a clever and efficient calculation in the other direction, finding u_{rr} and $u_{\theta\theta}$ in terms of u_x, u_y, x_r, etc.)

Problem L. Do §4.4, p. 119, #2, with the following modifications. Change the square to a rectangle $0 < x < a$, $0 < y < b$, and change the boundary condition along $y = 0$ to $u(x, 0) = 1$ for $0 < x < a/2$ and $u(x, 0) = 0$ for $a/2 < x < a$.

Problem M. The neutron flux in a nuclear reactor diffuses in the same way that heat does, except that new free neutrons are generated by fission at a rate proportional to the density u of neutrons. Thus u satisfies the PDE $u_t = k(\nabla^2 u + B^2 u)$, where the "buckling constant" B^2 depends on the mixture of fissionable and moderating material in the core.

(a) Find a series solution for the neutron density in a spherically symmetric reactor core with radius c if the density is zero at the edge of the core. (Hints: Be sure to use the Laplacian in *spherical* coordinates, see p. 406. The change of variable $v(r, t) = r\,u(r, t)$ is helpful. Prove that if u is continuous at $r = 0$, then $v(0, t) = 0$.)

(b) Show that if $B = \pi/c$, then the density approaches a non-zero steady state.

(c) What happens to the solution when $B > \pi/c$? What is the practical significance of this to you if you are working at the reactor?

Problem N. §4.4, p. 120, #7 with the following modifications: set $\beta = \pi$ and $r_0 = e^{-2}$, and let

$$g(r) = 5\sin(4\pi \log r) \quad \text{and} \quad h(r) = 3.$$

Write your final answer as simply and explicitly as possible.

Problem O. The purpose of this problem is to get you started on a problem that serves as a motivation for the work we will be doing in class on §§5.1-5.4. The problem is §5.5, p. 157, #4, but for now we will only get part way through it. You do *NOT* need to read anything in chapter 5 to answer the questions below. Save a copy of your work on part (b), because all part (b) will be assigned in the next assignment.

(a) Do part (a) of the problem - that is, find a steady state solution $v(r)$ - and state the problem for the transient solution $w(r, t)$ such that $u(r, t) = v(r) + w(r, t)$.

Where did z and θ go, you ask? All of the given information is symmetric around the z-axis, so there is no θ dependence. With the cylinder insulated on the top and bottom and no z dependence in the given data, the solution will also be independent of z.

(b) Apply separation of variables to the problem for $w(r, t)$. Notice we don't have the boundary conditions for a regular Sturm-Liouville problem. Are there are any restrictions on $R(r)$ at $r = 0$? (Don't worry about solving the equations until the next assignment.)