Due by noon Thursday, December 13, to my office or mailbox. Submit solutions for all five problems from part A below and two problems of your choice from part B below. All references to "Problem n-m" are from our text [ITM].

You may use your course notes and text [ITM], and may ask me questions. Do not consult any other sources (books, websites, people, etc.). Any clarifications given to one student that seem likely to be helpful to others will be e-mailed to everyone, so check your e-mail regularly as long as you are working on the exam. **Office hours during exam week**: M 3-4, TuWTh 11-12, and by appointment.

The usual rules and points for writing will apply. Start each problem on a new page, write neatly, and leave margins on all four edges of the page. Staple your solutions in order by problem number (A1, A2, ... Bn), with a cover sheet that lists your name and all the problems you are submitting. You may quote without proof any result or exercise in Chapters 1-12 of [ITM], or in any problem assigned for homework (including "work for yourself" problems but not "at least read" ones).

Total number of points, including writing points: 65.

A. Do all five of these problems.

A1. (5 points) Give an example of a retract that is not closed. Justify in detail: define the retraction and prove it is continuous, and prove the retract is not closed.

A2. (10 points) Let G be a topological group.

(a) Prove that if G is connected and U is a neighborhood of the identity, then every element of G can be written as a finite product of elements of U.

(b) Suppose that G is locally connected, and let G_0 be the connected component of the identity element. Prove that G_0 is a subgroup of G, that it is the only connected open subgroup of G, and that each connected component of G is homeomorphic to G_0.

Remarks, not part of the problem. Next quarter, we will work with Lie groups, which are groups that are also smooth manifolds such that the group operations are smooth. The results of this problem carry over from the category of topological groups to the category of Lie groups (morphisms = smooth maps) with the same proofs.

A3. (8 points) Problem 10-13.

A4. (10 points) Problem 11-7.

A5. (7 points) Problem 12-4. Be sure to justify that you have found the universal covering space of X.

B problems overleaf.

B. Do two of the following five problems.

B1. (10 points) Problem 10-9. (If you want to follow the hint and use the results of Problem 4-12, you must give the solution for that problem. The full strength of Problem 4-12 is not needed: It asks you for homeomorphisms where homotopy equivalences will do.)

B2. (10 points) Problem 10-21.

B3. (10 points) Problem 11-15. For part (a), use the generators for $\pi_1(X_2, 1)$ described in Example 11.17.

B4. (10 points) Problems 11-19 and 11-20.

B5. (10 points) Problems 12-14 and 12-15.

Brain Teaser, *not part of the final exam*, to ponder over quarter break if you wish.
Let H be the "Hawaiian Earring" described in Problem 12-15, with base point at the origin. The wedge sum of two copies of H does not satisfy the hypotheses of Theorem 10.7. (Why not?) Does the conclusion of the theorem nonetheless still hold for this example? Also consider the same question for the wedge sum of two copies of the cone over H.