

MATH 309B
FINAL REVIEW
June 4, 2008

1. Find solutions to the following inhomogeneous linear system of ODEs using all three methods discussed in class.

$$\mathbf{x}' = \begin{pmatrix} 4 & 1 \\ -2 & 1 \end{pmatrix} \mathbf{x} + \begin{pmatrix} e^t \\ e^{-t} \end{pmatrix}$$

2. Find the general solution to

$$\mathbf{x}' = \begin{pmatrix} 3 & \beta \\ 1 & -1 \end{pmatrix} \mathbf{x}$$

when $\beta = -5, -4, 0$. In each case, find the solution whose initial value at $t = 0$ is

$$\mathbf{x}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

3. Let $\Omega = \{(x, y) | 0 \leq x \leq a, 0 \leq y \leq b\}$. Use separation of variables to find solutions to the following (partial) Dirichlet problem:

$$\begin{aligned}\Delta u &= 0 \text{ in } \Omega \\ u(x, 0) &= 0 = u(x, b) \\ u(a, y) &= 0.\end{aligned}$$

Use these to solve the following Dirichlet problem:

$$\begin{aligned}\Delta u &= 0 \text{ in } \Omega \\ u(x, 0) &= 0 = u(x, b) \\ u(0, y) &= 1 = u(a, y).\end{aligned}$$

4. Compute the Fourier series for the following functions.

- (a) $\sin(x)$ on $(-\pi, \pi)$,
- (b) x^2 on $(-\pi, \pi)$,
- (c) e^x on $[0, \pi]$ extended to $(-\pi, \pi)$ evenly,
- (d) the step function

$$f(x) = \begin{cases} 0 & -\pi < x < -\pi/2, \pi/2 < x < \pi \\ 1 & -\pi/2 \leq x \leq \pi/2 \end{cases}$$

on $(-\pi, \pi)$.

5. Consider a string of length π stretched taut and attached at the ends to frictionless rods. Suppose the string lies with displacement zero when someone strikes it, giving it a velocity, at position x , of x^2 . Find the vertical displacement of the string as a function, $u(t, x)$, of time t and position x along the string. Write the solution as a linear combination of left and right moving waves.

6. Let $u(t, x)$ be a function with $t \geq 0$ and $0 \leq x \leq \pi$. Solve the following heat equation problem:

$$\begin{aligned}u_{xx} &= u_t \\u(t, 0) &= \pi, \quad u(t, \pi) = 2\pi \\u(0, x) &= \cos(x).\end{aligned}$$