Problem 2.(20 points)

W is a subspace of $\mathbb{R}^4$ and $\left\{ \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 4 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ is a basis for W.

(a) Find an orthogonal basis for W.

8

$$u_1 = \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \qquad u_2 = \begin{bmatrix} 2 \\ 0 \\ 4 \\ 0 \end{bmatrix} - \frac{\begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 0 \\ 4 \\ 0 \end{bmatrix}}{\begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}} \begin{bmatrix} -1 \\ 0 \\ +1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 0 \\ 4 \\ 0 \end{bmatrix} - \frac{2}{2} \begin{bmatrix} -1 \\ 0 \\ +1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 3 \\ 0 \end{bmatrix}$$

$\{\vec{u}_1, \vec{u}_2, \vec{u}_3\}$ is an orthogonal basis for W.

$$u_3 = \begin{bmatrix} -2 \\ 0 \\ 0 \\ 1 \end{bmatrix} - \frac{\begin{bmatrix} -2 \\ 0 \\ 0 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}}{\begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}} \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} - \frac{\begin{bmatrix} -2 \\ 0 \\ 0 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 0 \\ 3 \\ 0 \end{bmatrix}}{\begin{bmatrix} 3 \\ 0 \\ 3 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 0 \\ 3 \\ 0 \end{bmatrix}} \begin{bmatrix} 3 \\ 0 \\ 3 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -2 \\ 0 \\ 0 \\ 1 \end{bmatrix} - \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} - \frac{(-6)}{18} \begin{bmatrix} 3 \\ 0 \\ 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

4 **(b)**Is your basis in **(a)** orthonormal? Why or why not?

No b/c $\|\vec{u}_1\|^2 = 1^2 + 1^2 = 2 \neq 1$.