

Math 308C, Quiz 1

January 12, 2007

Name:

Show your work! Write clearly and legibly. Messy answers that I cannot understand will be assumed to be wrong.

**Problem 1.** Give an example of a  $3 \times 3$  linear system of equations which has infinitely many solutions. How do you know it has infinitely many solutions? Also give the augmented matrix associated to this linear system.

**Answer.**

$$\begin{aligned}x_1 &= 0 \\2x_1 &= 0 \\x_2 + x_3 &= 0\end{aligned}$$

Any consistent redundant system will work here. In particular, any homogeneous linear system of equations with a nontrivial solution will suffice. There are several ways to say this is a linear system of equations with infinitely many solutions. One way is to show 2 different solutions. Since a consistent system has exactly one solution or infinitely many, a system with at least two solutions has infinitely many solutions. Two solutions of this system are  $x_1 = x_2 = x_3 = 0$  and  $x_1 = 0, x_2 = 1, x_3 = -1$ . The augmented matrix associated to this linear system of equations is

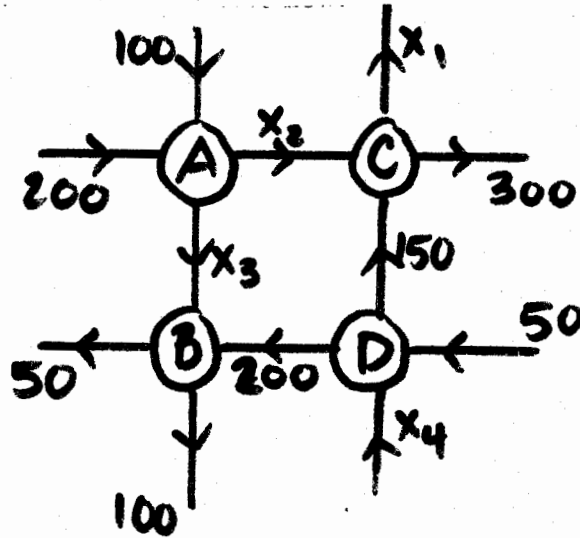
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}.$$

**Problem 2.** Given  $A = \begin{bmatrix} 1 & 0 \\ 4 & 2 \\ -1 & 1 \end{bmatrix}$  and  $B = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ , find  $AB$  if it is defined. Otherwise, state why  $AB$  is undefined.

**Answer.**  $A$  is a  $3 \times 2$  matrix and  $B$  is a  $2 \times 1$  matrix. The number of columns of  $A$  is the same as the number of rows of  $B$ , thus their product,  $AB$ , is defined and will be a  $3 \times 1$  matrix.

$$AB = \begin{bmatrix} 1 & 0 \\ 4 & 2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 16 \\ -1 \end{bmatrix}$$

**Problem 3.** Given the traffic network below, find a system of linear equations which describes the traffic flow.



**Answer.**

$$\begin{aligned}x_2 + x_3 &= 300 \\x_3 &= -50 \\-x_1 + x_2 &= 150 \\x_4 &= 300\end{aligned}$$

**Problem 4.** Which matrix (or matrices) is (are) in reduced echelon form?

a.  $\begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$

b.  $\begin{bmatrix} 1 & 3 & 4 \\ 0 & 0 & 1 \end{bmatrix}$

c.  $[1 \ -1 \ 5 \ 49]$

d.  $\begin{bmatrix} 1 & -2 & 5 \\ 0 & 1 & 8 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

e. none of the above

**Answer.** The only matrix in reduced echelon form is c. All the others either violate the condition that all zero rows are at the bottom or that all leading ones are the only nonzero entries in their columns.