

Name: _____

Math 308E Midterm II

31 July 2006

Instructions:

- This is a closed book exam. Graphing calculators are not allowed.
- You may use one $8\frac{1}{2}$ " \times 11" sheet of notes.
- Show your work! If you don't show your work, I won't be able to give you partial credit.
- Use the backs of pages if you run out of space.
- Good luck!

Problem 1	10 points	
Problem 2	10 points	
Problem 3	6 points	
Problem 4	9 points	
Problem 5	8 points	
Problem 6	7 points	
Total	50 points	

1. (10 points) Suppose A is the following (3×5) matrix:

$$A = \begin{bmatrix} 1 & 3 & 0 & -1 & 2 \\ -2 & -6 & 2 & 4 & 0 \\ 1 & 3 & 1 & 0 & 3 \end{bmatrix}$$

(a) (4 points) Find a basis for $\mathcal{R}(A)$, the range of A .

(b) (4 points) Find a basis for $\mathcal{N}(A)$, the null space of A .

(c) (2 points) Give an example of a 3×5 matrix that is row equivalent to A but has a *different* range (or tell me why this is not possible).

2. (10 points) True/False: Mark a 'T' or 'F' beside each statement. *No explanation is necessary.*

(a) If A is an 4×5 matrix with $\mathcal{R}(A) = \mathbb{R}^4$, then $\mathcal{N}(A) = \{\mathbf{0}\}$.

(b) If A and B are row equivalent matrices, then A and B must have the same **null space**.

(c) If C is a 4×3 matrix, then the nullity of C is at least 1.

(d) If W is a subspace of \mathbb{R}^n and $r\mathbf{x}$ is in W (for some vector \mathbf{x} and some scalar r), then \mathbf{x} must also be in W .

(e) If D is an $n \times n$ matrix whose columns form an orthogonal set, then D is necessarily nonsingular.

3. (6 points) Determine whether each of the following subsets of \mathbb{R}^3 is a *subspace* or not. If it is, no explanation is necessary. If it is not, just indicate which condition it fails.

(a) $W = \left\{ \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} : x_3 = 1 \right\}$

(b) $W' = \left\{ \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} : x_1 = 2x_3 \text{ and } x_2 = -x_3 \right\}$

(c) $W'' = \left\{ \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} : x_3 = x_1^2 + x_2^2 \right\}$

4. (9 points) Suppose C is a 4×5 matrix, and suppose $\mathcal{R}(C)$ (the range of C) has a basis given by

$$S = \left\{ \begin{bmatrix} 1 \\ 0 \\ -2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix} \right\}$$

(a) (2 points) Do the columns of C form a basis for $\mathcal{R}(C)$? Explain briefly.

For parts b) and d), suppose $\mathbf{d} = \begin{bmatrix} 1 \\ -2 \\ 0 \\ 3 \end{bmatrix}$

(b) (2 points) How many solutions are there to the system $C\mathbf{x} = \mathbf{d}$? Explain how you knew that.

(c) (3 points) How big is the matrix $C^T C$? Show that $C^T C$ is singular.

(d) (2 points) How many solutions are there to the system $C^T C \mathbf{x} = C^T \mathbf{d}$? Again, explain briefly.

5. (7 points) Suppose T is the following 2×2 matrix.

$$T = \begin{bmatrix} 1/2 & \sqrt{3}/2 \\ \sqrt{3}/2 & -1/2 \end{bmatrix}$$

(a) (3 points) Show directly that T is an *orthogonal transformation* (i.e., that $\|T\mathbf{v}\| = \|\mathbf{v}\|$ for all vectors \mathbf{v} in \mathbb{R}^2).

(b) (2 points) As a map from \mathbb{R}^2 to itself, what does T do geometrically?

(c) (2 points) Find *nonzero* vectors \mathbf{v}_1 and \mathbf{v}_2 such that $T\mathbf{v}_1 = \mathbf{v}_1$ and $T\mathbf{v}_2 = -\mathbf{v}_2$.

6. (8 points) Suppose B is some 3×4 matrix, and we know that the vector $\mathbf{b} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$ is *not* in the range of B , but every vector orthogonal to \mathbf{b} is in the range of B .

(a) (4 points) How many solutions are there to each of the following systems? Explain briefly how you knew.

$$B\mathbf{x} = \begin{bmatrix} 3 \\ 0 \\ 6 \end{bmatrix}$$

$$B\mathbf{x} = \begin{bmatrix} -4 \\ 5 \\ 2 \end{bmatrix}$$

$$B\mathbf{x} = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$$

(b) (2 point) What is the rank and nullity of B ? Explain how you knew.

(c) (2 points) Find a basis for $\mathcal{R}(B)$, the range of B .

EXTRA CREDIT (0 points) What is the plural form of the word 'matrix'?