Math 308E review questions for Midterm #2

- Given a matrix A, what is the definition of its null space, $\mathcal{N}(A)$? The definition of its range space, $\mathcal{R}(A)$? The definition of its row space, $\text{RowSp}(A)$?
- Given a subspace W, what is a spanning set for W? What is a basis for W?
- Given a basis for a subspace W, in how many ways can an arbitrary vector $\mathbf{x} \in W$ be written as a linear combination of basis vectors? Why?
- Given a subspace W of the form $W = \{\mathbf{x} : \mathbf{x} \text{ satisfies some system of linear equations}\}$, how can you find a basis for W?
- Given a matrix A, how can you find a basis for its null space, $\mathcal{N}(A)$?
- Given a matrix A, how can you find a basis for its range space, $\mathcal{R}(A)$? (There are at least three distinct methods.)
- Given a matrix A, how can you find a basis for its row space, $\text{RowSp}(A)$?
- What is the null space of a nonsingular matrix? What is its range space?
- Given a subspace W, what is its dimension?
- Given a matrix A, what is $\text{rank}(A)$? What is $\text{nullity}(A)$? How are the two related?
- Given a matrix A, how are $\text{rank}(A)$ and $\text{rank}(A^T)$ related?
- Given a subspace W with $\dim(W) = p$, what do you know about sets of vectors in W containing $p+1$ or more vectors? Containing fewer than p vectors? Containing p linearly independent vectors? Containing p vectors which span W?
- If U and V are subspaces and $U \subset V$, how are $\dim(U)$ and $\dim(V)$ related? If $\dim(U) = \dim(V)$, what does that imply about U and V, if anything?
- What does it mean for two vectors to be orthogonal? What does it mean for a *set* of vectors to be orthogonal?
- If S is an orthogonal set of vectors, is it necessarily linearly independent? What if S is an orthogonal set of *nonzero* vectors?
- What is an orthogonal basis for a subspace W? What is an orthonormal basis for W?
- Given a linearly independent set of vectors, how can you obtain an orthogonal set with the same span?
- Given an orthogonal set of nonzero vectors, how can you obtain an orthonormal set with the same span?
- What are the *coordinates* of a vector $\mathbf{v}$ with respect to a basis $\{\mathbf{u}_1, \ldots, \mathbf{u}_p\}$? If the basis is orthogonal, how can you easily compute these coordinates? If it's orthonormal?

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- What is the definition of a linear transformation? Given two subspaces V and W and a function $T : V \to W$ between them, how can you determine whether or not T is a linear transformation?

- What is the null space $\mathcal{N}(T)$ of a linear transformation T? What is the range space $\mathcal{R}(T)$? What are the rank and nullity of T?

- Is it necessarily true that for any linear transformation T, there is some matrix A such that T is given by $T(\mathbf{x}) = A\mathbf{x}$ for all vectors $\mathbf{x}$?

- Suppose you know that $T(\mathbf{x}) = A\mathbf{x}$ for some $(m \times n)$ matrix A. Given $T(\mathbf{e}_1), \ldots, T(\mathbf{e}_n)$, how can you find A? Given a basis $\{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$ for $\mathbb{R}^n$ and $T(\mathbf{v}_1), \ldots, T(\mathbf{v}_n)$, how can you find A?

- What is an orthogonal transformation on $\mathbb{R}^2$? What is the matrix representation of a rotation through an angle of θ on $\mathbb{R}^2$?

- What property must an $(n \times n)$ matrix satisfy in order to be called an orthogonal matrix? What do you know about the columns of such a matrix?

- What is a least-squares solution to a linear system $A\mathbf{x} = \mathbf{b}$? What do you know about the least squares solution if the linear system is inconsistent? If it is consistent?

- When is a least-squares solution unique?

- How can you find the least-squares linear fit to a given data set? What about a least-squares quadratic fit?

- Can you list at least *five* things that are equivalent to the statement that a matrix A is nonsingular?