PRACTICE PROBLEMS

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(1) If $w = [6,3,1,7,5,2,4]$, what is w^{-1}? How would you denote these two permutations in each of the different ways to see a permutation?

(2) Recall the (Rothe) diagram of a permutation $w \in S_n$ is defined to be $D^*(w) = \{(i,j) \ : \ i < w(j) \text{ and } w^{-1}(i) > j\} \subset [n] \times [n]$ drawn in matrix coordinates. Define the code of w to be $(c_1, c_2, \ldots, c_n)$ where c_j is the number of elements in $D^*(w)$ in column j. List out the permutations in S_n in lexicographic order. Let $N(w)$ be the position of w in this list. Then

$$N(w) = c_1(n-1)! + c_2(n-2)! + \ldots + c_{n-1}1!.$$

(3) Prove $\prod_{k=1}^{n-1}(1+x+x^2+\cdots+x^k) = \sum_{w\in S_n} x^{inv(w)}$ where $inv(w)$ is the number of inversions of w, or equivalently pairs $i<j$ such that $w_i > w_j$.

(4) Prove $\prod_{k=1}^{n-1}(1+x+x^2+\cdots+x^k) = \sum_{w\in S_n} x^{maj(w)}$ where $maj(w)$ is the sum of the number of the descents for w. For example $w = [6,3,1,7,5,2,4]$ has descent set $\{1,2,4\}$ so $maj(w) = 1+2+4 = 7$.

(5) If $\pi \in S_n$ maps to (P,Q) under RSK, then $Des(\pi) = Des(Q)$.

(6) Write out $s_\mu(x_1,x_2,x_3)$ for all partitions μ of size 4 and 5.

(7) Compute the number of standard tableaux of shape $(k, k-1, k-2, \ldots, 1)$ for some small values of k.

(8) Let T be a tree with n vertices. What is the coefficient of $m_{(2,1,1,\ldots,1)}$ in the expansion of the chromatic symmetric function X_T?

(9) Expand $s_{(4,2)}$ in fundamental quasisymmetric functions.

(10) Find a rule to multiply monomial quasisymmetric functions $M_\alpha M_\beta$ and expand in the basis of monomial quasi's again.

(11) Find a rule to multiply monomial quasisymmetric functions $F_\alpha F_\beta$ and expand in the basis of fundamental quasi's again.

(12) Expand the chromatic symmetric function of a graph G into fundamental quasisymmetric functions using acyclic orientations.

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