PHILOSOPHY OF PROBABILITY AND ITS RELATIONSHIP (?) TO STATISTICS

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Is this a real quote?

“Probability does not exist. We can say nothing about the probability of death of an individual even if we know his condition of life and health in detail.”

“Probability does not exist.”

Bruno **de Finetti**, the most prominent representative of the “**subjective philosophy**” of probability

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“We can say nothing about the probability of death of an individual even if we know his condition of life and health in detail.”

Richard **von Mises**, the most prominent representative of the “**frequency philosophy**” of probability

Four mature philosophies

Created in twentieth century

Name	**Principal philosopher**	**What is the nature of probability?**
Logical	Rudolf Carnap	Weak implication
Propensity	Karl Popper	Physical property
Frequency	Richard von Mises	Attribute of a sequence
Subjective	Bruno de Finetti	Personal opinion

Frequency interpretation of probability

(i) Data: Boys are born with frequency 0.513

(ii) Law of Large Numbers

Subjective interpretation of probability

(i) Use mathematical probability to express uncertainty

(ii) Given new information (data), update your opinion using the Bayes Theorem

(iii) Make decisions that maximize the expected gain (utility)

None of the above ideas was invented by von Mises or de Finetti.

The fundamental claim of both frequency and subjective philosophies of probability:
"**It is impossible to measure the probability of an event**."

How about repeated observations?

Von Mises: Probability is a measurable attribute of a sequence. Tigers are aggressive. Aggressiveness is not an attribute of atoms in tigers' bodies.

De Finetti: Observed frequency does not falsify a prior probability statement because it is based on different information.

The fundamental claim of both frequency and subjective philosophies of probability:
"**It is impossible to measure the probability of an event**."

Motivation?

One needs to limit scientific applications of probability theory. "Work" in everyday parlance is not the same as "work" in physics.

Smoking gun: Absence of relevant discussion.

Why should we use probability?

- Von Mises: Apply mathematical probability theory to observable frequencies in “collectives” (i.i.d. sequences).
- De Finetti: Use mathematical probability theory to coordinate decisions.

Weaknesses of the two theories

- The domain of applicability is more narrow than the actual scientific applications of probability.
- Von Mises' collectives and de Finetti's decision theoretic approach are unusable.

Von Mises' collectives

A collective is a sequence of experiments or observations such that the frequency of a given event is the same (in the limit) along every subsequence chosen without prophetic powers.

Why use collectives rather than i.i.d. sequences?

$$A_1, A_2, \ldots$$

$$P(A_1) = P(A_2)$$

Hypothesis testing

- Routine hypothesis testing
- Scientific hypothesis testing

Von Mises: Elements of a collective have **everything** in common except probability.

Hypothesis testing: Elements of a sequence of tests have **nothing** in common except probability.

Contradictions in von Mises' book

Hypothesis testing in von Mises' book: Bayesian approach.

Frequency interpretation of results: conditioning on the data.

The corresponding collective is imaginary.

$$P(\textit{observing identical data}) \approx 10^{-100}$$

"The implication of Germany in a war with the Republic of Liberia is not a situation which repeats itself."

$$P(\textit{observing } G-L \textit{ collective}) \approx 10^{-100}$$

Unbiased estimators

- Frequency interpretation requires a long sequence of “identical” data sets.
- Why not combine all the data sets into one data set?

SUBJECTIVE PHILOSOPHY	BAYESIAN STATISTICS
"Subjective" = "does not exist"	"Subjective" = "informally assessed"
All probabilities are subjective.	Some probabilities are subjective.
$P_1(A \mid B) \neq P_2(A \mid B)$	$P(A \mid B_1) \neq P(A \mid B_2)$
Probabilities are used to coordinate decisions.	There are no decisions to coordinate.

Main philosophical ideas of de Finetti

- You can achieve a deterministic goal using probability calculus.
- You do not need to know the real (objective) probabilities to achieve the deterministic goal, whether these probabilities exist or not.

The Black-Scholes theory (arbitrage pricing) is the only successful application of de Finetti's ideas.