RESONANCE

From Probability to Epistemology and Back

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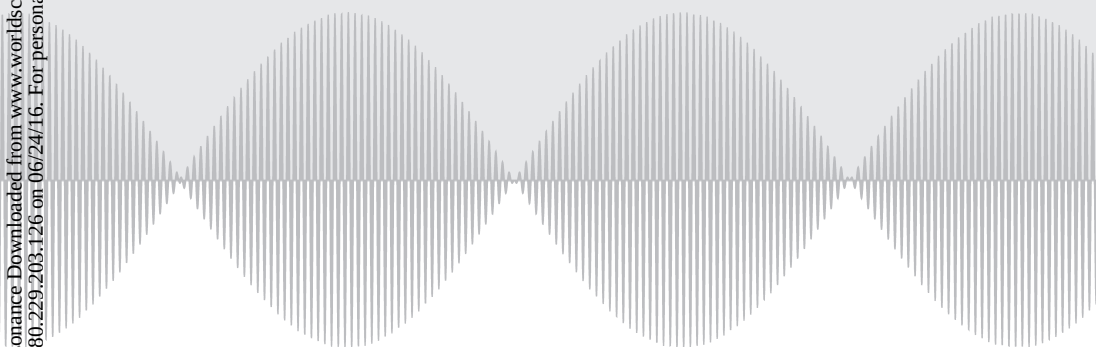

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To Agnieszka, my wife

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Preface

Do you believe that two and two makes four? Yes? Then why? You may reply that "It works" or "Everyone knows that $2 + 2 = 4$." Both answers are perfectly reasonable. We need this kind of attitude in everyday life. Any reasoning more formal than that would paralyze our normal activities. But the two common sense answers completely fail in some situations. This book is devoted to one of these situations — the philosophical controversy surrounding the concept of probability.

The philosophy of probability has several major branches, the best known of which are frequency and subjective. These two philosophies are (mistakenly) associated with the two main branches of statistics — frequency and Bayesian. The temperature of the intellectual dispute concerning the meaning and applications of probability has been high in both philosophy and statistics.

Why did philosophers and statisticians fail to check what "works?" They did not. They checked and they know very well what works except that representatives of each side of the dispute swear that their own methods "work" and those of their opponents do not. This controversy shows that we have to try to understand the sources of our knowledge at a level deeper than "It works" — this is what this book is about. My previous book, "The search for certainty," [Burdzy (2009)] codified the scientific laws of probability. All I did was to formalize what was already present in probability and statistics textbooks. In this volume, I will try to supply epistemological foundations for probability. One particular philosophical idea provides a strong motivation for my analysis — the idea that probability is subjective. In the previous book, I showed that the interpretation of subjectivism advocated by Bruno de Finetti made no sense whatsoever. But the subjectivist current in philosophy of probability

is not limited to the extravagant theory of de Finetti and it deserves an honest analysis. Contrary to the popular belief, de Finetti was an antisubjectivist because he denied any value to subjective probabilistic opinions. Paradoxically, my epistemological analysis of personal knowledge (I borrowed this phrase from [Polanyi (1958)]) will make me more of a subjectivist than I ever expected or wanted to be.

I will base my epistemology on the concept of "resonance," loosely related to its namesake in physics. I will use resonance in my analysis of the philosophical problem of induction. I will also briefly analyze three other well known philosophical problems of consciousness, intelligence and free will. It is not my intention to present detailed explanations for these phenomena but to propose a small set of ideas that will remove the mystery out of these seemingly incomprehensible aspects of human intellect. Going back to the original motivation for this project, I will use this epistemology to outline my opinions about the apparent subjectivity of probability.

I imported much of the discussion of philosophy of probability and its relationship to statistics from [Burdzy (2009)] but I changed the presentation of that material and I put it in a new context — that of the epistemological ideas developed in this volume.

I am grateful to people who offered their comments on [Burdzy (2009)] or the draft of this manuscript and thus helped me improve the book: Itai Benjamini, Erik Björnemo, Nicolas Bouleau, Arthur Fine, Adrew Gelman, Artur Grabowski, Peter Hoff, Wilfrid Kendall, Vlada Limic, Dan Osborn, Jeffrey Rosenthal, Jaime San Martin, Jim Pitman, Christian Robert, Pedro Terán, John Walsh, Larry Wasserman and anonymous referees.

Special thanks go to Janina Burdzy, my mother and a probabilist, for teaching me combinatorial probability 40 years ago. The lesson about the fundamental role of symmetry in probability was never forgotten.

I am grateful to Agnieszka Burdzy, my wife, for discussing with me a number of philosophical problems related to this book.

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Seattle, 2015

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