MIDTERM EXAM GUIDE

The exam will consist of 6 questions. The midterm is worth a total of 300 points and will covers the material in sections 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.9, 3.1, 3.2, 3.3, 3.4, 3.5, and 3.6 of the text. The content of each question and its point value is listed below. These questions may be multipart. You must show all of your work to get full credit for your solution. Except in the case of the True-False questions, an answer that appears with no accompanying work will be given zero credit.

Question 1: (50 points) In this question you will be asked to solve a linear system of equations by converting its associated augmented matrix to reduced echelon form using Gauss-Jordan elimination.

Question 2: (50 points) In this question you will be asked to compute the inverse of a matrix.

Question 3: (50 points) In this question you will be asked to either

(i) describe matrix-vector multiplication from both the row and column perspective, or

(ii) describe matrix-matrix multiplication from the column, row, and element wise perspectives,

or both.

Question 4: (50 points) In this question you will be asked to compute the Gauss-Jordan elimination matrix for given pivot in a given matrix, and then to multiply the given matrix on the left by this elimination matrix.

Question 5: (50 points) In this question you will be asked to compute a basis for one or more of the 4 fundamental subspaces associated with a matrix A: $\mathrm{Ran}(A)$, $\mathrm{Nul}(A)$, $\mathrm{Ran}(A^T)$, $\mathrm{Nul}(A^T)$. However, the subspace may be described as either the linear span of a collection of vectors or the subspace orthogonal to a collection of vectors.

Question 6: (50 points) This question is a collection of true-false questions based on the the given sections of the text.

SAMPLE QUESTIONS

1. Describe the solution set the following linear system in vector form by converting it to reduced echelon form:

$$\begin{array}{rcrcrcrcrcr} x_1 & & & + & x_3 & + & x_4 & - & 2x_5 & = & 1 \\ 2x_1 & + & x_2 & + & 3x_3 & - & x_4 & + & x_5 & = & 0 \\ 3x_1 & - & x_2 & + & 4x_3 & + & x_4 & + & x_5 & = & 1 \end{array}$$

2. Compute the inverse of the following matrices:

$$A = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 4 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 2 & 1 \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} 1 & 4 & 2 \\ 0 & 2 & 1 \\ 3 & 5 & 3 \end{bmatrix}$$

3. (a) Describe matrix-vector multiplication from both the row and column perspective.

   (b) Describe matrix-matrix multiplication from the column, row, and elementwise perspectives.

4. (a) Give the 4×4 Gauss-Jordan elimination matrix that converts the vector $v = \begin{pmatrix} -2 \\ 1 \\ 2 \\ -1 \end{pmatrix}$ into the unit coordinate vector e_3.

   (b) Compute the matrix multiplication AB, where the matrix A is the matrix obtained in part (a) and

$$B = \begin{bmatrix} 1 & 0 & -2 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & -1 & 3 \end{bmatrix}$$

   (c) Write the Gauss-Jordan elimination matrix G for the matrix B with pivot $B_{(2,3)} = 1$, where

$$B = \begin{bmatrix} 2 & 3 & 4 & 5 \\ 1 & 2 & 1 & 2 \\ 1 & 5 & -5 & 1 \end{bmatrix}.$$

   (d) Compute GB where G and B are as given in part (c).

5. Compute a basis for the 4 fundamental subspaces associated with the following matrices:

$$A = \begin{bmatrix} 1 & 0 & 1 & 1 & -2 \\ 2 & 1 & 3 & -1 & 1 \\ 3 & -1 & 4 & 1 & 1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 2 & 1 & -2 \\ 2 & 3 & 4 & 5 \\ 1 & 5 & -5 & 1 \end{bmatrix}$$

6. Answer the following true-false questions.

   (a) **T** - **F**: A linear system can have precisely 3 distinct solutions.

   (b) **T** - **F**: The matrix $M = \begin{bmatrix} b & 3 \\ b^2 & a \end{bmatrix}$ is nonsingular if and only if either $b = 0$ or $a = 3b$.

   (c) **T** - **F**: A homogeneous linear system of 5 equations and 6 unknowns always has a non-trivial solution.

   (d) **T** - **F**: If $B \in I\!R^n$ satisfies $B^3 + B^2 + B = I$, then B is nonsingular.

   (e) **T** - **F**: The rank of a matrix is equal to the number of its non-zero rows.

   (f) **T** - **F**: If the matrix A is row equivalent to B, then B is row equivalent to A.

   (g) **T** - **F**: Row equivalent matrices always have the same reduced echelon form.

   (h) **T** - **F**: There are 2^3 subspaces in $I\!R^2$.

   (i) **T** - **F**: Let $A, B \in I\!R^{n \times n}$. If B is nonsingular and AB is nonsingular, then A is nonsingular.

   (j) **T** - **F**: Let $A \in I\!R^{n \times n}$. If A^T is singular, the A cannot have an inverse.

   (k) **T** - **F**: If $B^2 + B + 2I = 0$, then B is singular.

   (l) **T** - **F**: If u, v, w are any vectors in $I\!R^4$, then the vectors $3u - 2v$, $2v - 4u$, and $4w - 3u$ are linearly dependent.

   (m) **T** - **F**: If $A = BC$, then every solution to $Cx = 0$ is a solution to $A = 0$.

   (n) **T** - **F**: Every set of 6 vectors in a 4 dimensional subspace of $I\!R^{12}$ is linearly dependent.

   (o) **T** - **F**: If $B = \{v^1, v^2, \ldots, v^n\}$ is a basis for $I\!R^n$ and W is a subspace of $I\!R^n$, then some subset of B is a basis for W.

   (p) **T** - **F**: If W_1 and W_2 are subspaces of $I\!R^n$ of dimension $k < n$ with $W_1 \subset W_2$, then $W_1 = W_2$.