FINAL EXAM OUTLINE FOR MATH 407 EXAM DATE: Thursday, August 18, 2011: 9:40am-11:40am

The final exam for this course is set to be given on Thursday, August 18 at 9:40am-11:40am in the same classroom that the course always meets in.

EXAM OUTLINE

The final exam will consist of 6 questions each worth 60 points except for problem 4 which is worth 50 points. The content of each question is as follows.

Question 1: In this question you will be asked to model one or more of the LP models 1–25 given on the class web page.

Question 2: In this question you will be given one or more LPs and asked to solve them. The solution method may or may not be specified (graphical, the primal simplex algorithm, the two phase simplex algorithm, the dual simplex algorithm).

You will need to show all of your work to get full credit. In addition, you may be asked to answer a question about the nature of the solution that you have found or the nature of the dual solution.

Question 3: In this question you will be asked to put a given LP into standard form.

Question 4: In this question you will be asked to formulate the dual of a given LP without first bringing it to standard form.

Question 5: In this question you will be given an LP and a vector. You will then be asked to determine if the vector solves the LP using either the Complementary Slackness Theorem or the Geometric Duality Theorem.

Question 6: In this problem you will be given an LP model, its initial tableau, and an associated optimal tableau. You will then be asked to answer certain questions about the problem using the techniques of sensitivity analysis.

SAMPLE QUESTIONS

1. A farmer has to purchase the following quantities of fertilizer from four different shops, subject to the following capacities and prices. How can he fulfill his requirements at minimal cost?

Fertilizer Type	*Minimum Required (tons)*
1	185
2	50
3	50
4	200
5	185

Shop Number	*Maximum (all types combined) They Can Supply*
1	350 tons
2	225
3	195
4	275

	Price in Money Units per Ton of Fertilizer Type				
At Shop	1	2	3	4	5
1	45.0	13.9	29.9	31.9	9.9
2	42.5	17.8	31.0	35.0	12.3
3	47.5	19.9	24.0	32.5	12.4
4	41.3	12.5	31.2	29.8	11.0

2. (a) Solve the following LP stating its solution and optimal value.

$$\begin{array}{lrcrcrcrcl} \text{maximize} & 4x_1 & + & 4x_2 & + & 5x_3 & + & 3x_4 & & \\ \text{subject to} & x_1 & + & x_2 & + & x_3 & + & x_4 & \leq & 40 \\ & x_1 & + & x_2 & + & 2x_3 & + & x_4 & \leq & 40 \\ & 2x_2 & + & 2x_2 & + & 3x_3 & + & x_4 & \leq & 60 \\ & 3x_1 & + & 2x_2 & + & 2x_3 & + & 2x_4 & \leq & 50 \\ & 0 & \leq & x_1, & & x_2, & & x_3, \quad x_4. & & \end{array}$$

   (b) State the dual of this LP and give its solution.

3. Put the following LP in standard form.

$$\begin{array}{lrcrcrcr} \text{minimize} & & - & x_2 & + & x_3 & & \\ \text{subject to} & x_1 & & & - & 4x_3 & \geq & -5 \\ & -3x_1 & + & x_2 & & & = & -3 \\ & x_1 & + & x_2 & + & x_3 & \leq & 10 \\ & x_1 \geq & -1 & , & 0 \geq & x_2 & & \end{array}$$

4. Formulate a dual for the following LPs.

   (a)

$$\begin{array}{lrcl} \text{minimize} & c^T x & & \\ \text{subject to} & Ax & \leq & 0 \\ & Bx & = & 0, \end{array}$$

   where $c \in \mathbb{R}^n$, $A \in \mathbb{R}^{s \times n}$, and $B \in \mathbb{R}^{t \times n}$.

(b)

$$\begin{array}{ll} \text{maximize} & 2x_1 - 3x_2 + 10x_3 \\ \text{subject to} & x_1 + x_2 - x_3 = 12 \\ & x_1 - x_2 + x_3 \leq 8 \\ & 0 \leq x_2 \leq 10 \end{array}$$

5. Use either the Complementary Slackness Theorem or the Geometric Duality Theorem to determine is the vector $x = (0, 5, 0, 1, 1)^T$ solves the LP

$$\begin{array}{lrcrcrcrcrcr} \text{maximize} & & x_2 & & & + & 5x_4 & + & 5x_5 & & & \\ \text{subject to} & x_1 & + & 2x_2 & - & x_3 & + & x_4 & & & \leq & 11 \\ & 3x_1 & + & x_2 & + & 4x_3 & + & x_4 & + & x_5 & \leq & 10 \\ & 2x_1 & - & x_2 & + & 2x_3 & + & x_4 & + & 2x_5 & \leq & -2 \\ & x_1 & & & & & + & x_4 & + & 3x_5 & \leq & 4 \\ & 0 & \leq & x_1, & x_2, & x_3, & x_4, & x_5 & & & & \end{array}$$

6. **ARTY'S TIE–DYED T–SHIRTS:** Arty Binewski is making his plans for the Fremont Fair where he sells four types of custom made sweatshirts. These types are

   (a) tie-dyed,

   (b) single dyed with a silkscreen caption,

   (c) single dyed with a silkscreen design, and

   (d) single dyed and hand painted with silkscreen paint.

   Arty makes these sweatshirts in batches of 20 shirts and has bought 400 white sweatshirts at Costco for \$7 a shirt. He figures that he has on hand enough dye for 30 batches of single dyed shirts. However, the tie-dyed shirts require 3 dyings. This dye is top quality stuff and each shirt costs about \$.75 for a single dye. Arty also has on hand enough silkscreen paint for 25 batches of the type (b) sweatshirts, however, the type (c) and (d) sweatshirts require about twice as much paint per shirt. The silkscreen paint he uses is also quite expensive and Arty estimates that each type (b) shirt requires about \$0.2 worth of paint. Between now and the time the Fremont fair opens Arty estimates that he'll have about 50 hours to devote to this project. Being a linear programming fanatic, he decides to determine his production by solving an appropriate linear program to maximize his profit. The initial and final tableaus for this LP follow.

	a	b	c	d	s_1	s_2	s_3	s_4		
shirts	1	1	1	1	1	0	0	0	|	20
paint	0	1	2	2	0	1	0	0	|	25
labor	2	1	2	3	0	0	1	0	|	50
dye	3	1	1	1	0	0	0	1	|	30
	180	160	200	240	0	0	0	0	|	0

   where the variables a, b, c, and d represent the number of 20 shirt batches for each of the 4 types of shirts (a), (b), (c), and (d), respectively. Thus, in particular, the cost coefficients are in dollars of profit per batch of 20 shirts (e.g. each type (a) shirt produces a profit of \$9). The optimal tableau is as follows:

1	0	0	0	$-1/2$	0	0	$1/2$	|	5
0	0	-1	0	$5/2$	-2	1	$-3/2$	|	5
0	1	0	0	3	-1	0	-1	|	5
0	0	1	1	$-3/2$	1	0	$1/2$	|	10
0	0	-40	0	-30	-80	0	-50	|	$-4,100$

Answer each of the following questions as if it were a separate event. Do not consider the cumulative effects between problems unless explicitly requested to do so. Clearly label all of your final solutions and show all of your work. Partial credit can only be assigned if there is a clear line of reasoning so try to be neat.

a) Arty would like to produce at least a few of each type of shirt. At what price should he sell the type (c) shirts in order to make it efficient to produce at least a few of them?

b) Arty is also considering the possibility of making some batik sweatshirts. Just as for the tie-dyes these require a three dye process, they use no paint, but do require about 3 hours of work for each batch of 20 shirts. If he sells these shirts for $19.25 each, what is the new production schedule?

c) Should Arty buy more dyes and paints for his sweatshirts? If so, how much should he spend on each (dyes and/or paints) in order to maximize the return on this investment? (Justify your answer! A single pivot, or at least a partial pivot, is required to answer this question.)