

January 11, 2008

There are 2 problems. Stop now and make sure you have both problems. If you do not have them both, then request a new quiz. The first problem is worth 30 points and the second is worth 45 points for a total of 75 points. Show all of your work and follow the directions provided. Partial credit will be given for partial solutions. *CALCULATORS ARE NOT ALLOWED!*

Problem	Score
<u>1</u>	_____
<u>2</u>	_____
<u>Total</u>	=====

1. [a](15 points) Under what conditions is the set $S \subset \mathbb{R}^n$ a subspace of \mathbb{R}^n .

Solution S is a subspace of \mathbb{R}^n if

- (i) $0 \in S$
- (ii) $x + y \in S$ for all $x, y \in S$.
- (iii) $\alpha x \in S$ for all $\alpha \in \mathbb{R}$ and $x \in S$.

[b](15 points) Let A be a linear transformation from \mathbb{R}^n to \mathbb{R}^m with $m < n$. Give the definition for the *null space* of A and provide a simple lower bound for its dimension.

Solution

$$\text{Nul}(A) = \{x \in \mathbb{R}^n \mid Ax = 0\} \quad \text{and} \quad \dim(\text{Nul}(A)) = n - \dim(\text{Ran}(A)) \geq n - m$$

2. Consider the system

$$\begin{aligned} -x_1 & \quad \quad + 4x_3 = 200 \\ -x_1 + x_2 + 9x_3 & = 200 \\ 2x_1 - x_2 + 7x_3 & = 200 . \end{aligned}$$

(a)(15 points) Write the augmented matrix corresponding to this system.

Solution

$$\left[\begin{array}{ccc|c} -1 & 0 & 4 & 200 \\ -1 & 1 & 9 & 200 \\ 2 & -1 & 7 & 200 \end{array} \right]$$

(b)(20 points) Reduce the augmented system in part (a) to echelon form.

Solution

$$\begin{array}{ccc|c|c} -1 & 0 & 4 & 200 & \\ -1 & 1 & 9 & 200 & \\ 2 & -1 & 7 & 200 & \\ \hline 1 & 0 & -4 & -200 & -r_1 \\ 0 & 1 & 5 & 0 & r_2 - r_1 \\ 0 & -1 & 15 & 600 & r_3 + 2r_1 \\ \hline 1 & 0 & -4 & -200 & \\ 0 & 1 & 5 & 0 & \\ 0 & 0 & 20 & 600 & r_2 + r_3 \\ \hline \end{array}$$

(c)(10 points) Describe the set of solutions to the given system.

Solution

$$\begin{aligned} 20x_3 = 600 & \quad \rightarrow x_3 = 30 \\ x_2 + 5x_3 = 0 & \quad \rightarrow x_2 = -5x_3 = -150 \\ x_1 - 4x_3 = -200 & \quad \rightarrow x_1 = -200 + 4x_3 = -80 \end{aligned}$$

Therefore, the solution set consists of the unique point

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -80 \\ -150 \\ 30 \end{pmatrix} .$$