

# Contents

<b>0</b>	<b>Mathematical Preliminaries</b>	<b>3</b>
0.1	Norms . . . . .	3
0.1.1	Equivalence of Norms . . . . .	4
0.2	Open, Closed, and Compact Sets . . . . .	4
0.3	Continuity and the Existence of Extrema . . . . .	5
0.4	Dual Norms . . . . .	5
0.5	Operators . . . . .	6
0.5.1	Operator Norms . . . . .	6
0.5.2	Spectral Radius . . . . .	7
0.5.3	Condition number . . . . .	8
0.5.4	The Frobenius Norm . . . . .	8
0.6	Review of Differentiation . . . . .	8
0.6.1	The Implicit Function Theorem . . . . .	10
0.6.2	Some facts about the Second Derivative . . . . .	11
0.6.3	Integration . . . . .	11
0.6.4	More Facts about Continuity . . . . .	12
0.6.5	Some Facts about Symmetric Matrices . . . . .	14
<b>1</b>	<b>Optimality Conditions: Unconstrained Optimization</b>	<b>15</b>
1.1	Differentiable Problems . . . . .	15
1.2	Convex Problems . . . . .	17
1.3	Convex Composite Problems . . . . .	21
1.3.1	A Note on Directional Derivatives . . . . .	22
<b>2</b>	<b>Basic Convergence Theory</b>	<b>23</b>
2.1	Global Theory . . . . .	23
2.1.1	Line-Search Methods . . . . .	23
2.1.2	Trust-Region Methods . . . . .	28
2.2	Local Theory . . . . .	33
2.2.1	Strong Convexity . . . . .	34
2.2.2	Linear Convergence . . . . .	35
2.3	Newton's Method . . . . .	40

2.3.1	Newton's Method for Equation Solving . . . . .	40
2.3.2	Newton's Method for Minimization . . . . .	44
2.4	Linking Global and Local Methods . . . . .	45
2.4.1	Line Search Methods . . . . .	47
2.4.2	Trust-Region Methods . . . . .	50
<b>3</b>	<b>Conjugate Direction Methods</b>	<b>53</b>
3.1	General Discussion . . . . .	53
3.2	The Conjugate Gradient Algorithm . . . . .	57
3.3	Extensions to Non-Quadratic Problems . . . . .	58
<b>4</b>	<b>Matrix Secant Methods</b>	<b>61</b>
4.1	Equation Solving . . . . .	61
4.2	Minimization . . . . .	65
<b>5</b>	<b>Optimality Conditions: Constrained Optimization</b>	<b>73</b>
5.1	First-Order Conditions . . . . .	73
5.2	Regularity and Constraint Qualifications . . . . .	76
5.3	Second-Order Conditions . . . . .	79
5.4	Optimality Conditions in the Presence of Convexity . . . . .	80
5.5	Application to Solving Trust-Region Subproblems . . . . .	84
<b>6</b>	<b>LP's, QP's, and LCP's</b>	<b>89</b>
6.1	Introduction . . . . .	89
6.2	Boundedness Properties of LCP . . . . .	90
6.3	The Central Path . . . . .	95
6.3.1	Asymptotic behavior of the central path . . . . .	98
6.4	An Infeasible Interior Point Algorithm . . . . .	102
<b>7</b>	<b>The Gradient Projection Algorithm</b>	<b>109</b>
7.1	Projections and Optimality Conditions . . . . .	109
7.2	The Basic Gradient Projection Method . . . . .	111
7.3	The Computation of Projections . . . . .	113
<b>8</b>	<b>Exterior Penalty Methods</b>	<b>115</b>
8.1	Basic Theory . . . . .	115
8.2	Exact Penalization . . . . .	119
<b>9</b>	<b>The Method of Multipliers</b>	<b>125</b>
9.1	Introduction . . . . .	125
9.2	The Augmented Lagrangian . . . . .	126
9.2.1	<u>Algorithm: The Method of Multipliers</u> . . . . .	129