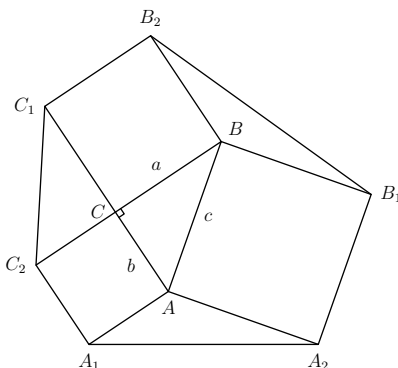


Challenge Of the Week

March 11—March 16, 2008

Problem:

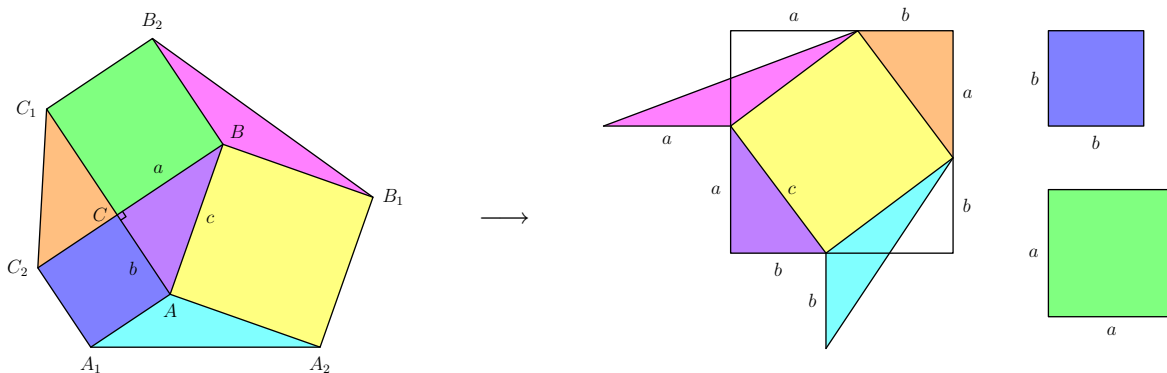
A right triangle ABC with side lengths a, b, c (with $c^2 = a^2 + b^2$) determines a hexagon whose vertices are the “outside” corners of the squares on the sides AB, BC and CA . Find the area of the hexagon in terms of a, b , and c .



Solution:

There are lots of ways to solve this, anything from trig to algebra to vector calculus works.

Here's a simple geometric solution. Rearrange the pieces:



We can see that the pink triangle BB_1B_2 has area $ab/2$, since it has base a and height b ; similarly, the cyan triangle AA_1A_2 also has area $ab/2$. Thus we can fit these triangles into the large $(a + b) \times (a + b)$ square (this may be done by breaking each of these triangles in half, as is clear from the picture), and so the entire area is $(a + b)^2 + a^2 + b^2$.