Challenge Of the Week

April 1—April 6, 2008

Problem:

Consider the following cryptarithm:

$$\begin{array}{r} \texttt{APRIL} \\ -\ \texttt{FOOLS} \\ \hline \texttt{DAY} \end{array}$$

Each letter A, P, R, I, L, F, O, S, D, and Y stands for a different digit $(0-9)$. After solving the cryptarithm, let $\texttt{UVW} = |\texttt{DAY} - \texttt{YAD}|$ (so that UVW is a 3-digit number, letting U and V be leading 0's if necessary). Here's the problem: what is $\texttt{UVW} + \texttt{WVU}$?

Solution:

The first part of the problem is the cryptarithm. There are six possible solutions:

APRIL	30685	30685	50683	50683	60281	60281
− FOOLS	− 29951	− 29954	− 49931	− 49932	− 59914	− 59917
DAY	734	731	752	751	367	364

However, in the spirit of an April Fool's puzzle, solving the cryptarithm was a red herring; we can solve the problem easily as follows:

Given the number $\texttt{DAY} = 100\texttt{D} + 10\texttt{A} + \texttt{Y}$, we compute

$$\texttt{UVW} = |\texttt{DAY} - \texttt{YAD}| = |(100\texttt{D} + 10\texttt{A} + \texttt{Y}) - (100\texttt{Y} + 10\texttt{A} + \texttt{D})| = 99|\texttt{D} - \texttt{Y}|.$$

Since we know that D and Y stand for different digits, $\texttt{UVW} \neq 000$, and so UVW is a nonzero, 3-digit multiple of 99, namely: 099, 198, 297, 396, 495, 594, 693, 792, 891, or 990. The middle digit is always 9, while the first and last digits add up to 9. Thus,

$$\texttt{UVW} + \texttt{WVU} = 100(\texttt{U} + \texttt{W}) + 10(\texttt{V} + \texttt{V}) + (\texttt{W} + \texttt{U}) = 900 + 10 \cdot 18 + 9 = 1089.$$