

Challenge of the Week

May 13–May 19, 2008

Problem:

Show that on every great circle around the Earth, there are two antipodal points that have the same temperature. (Assume that the temperature on Earth is a continuous function.)

Solution:

Given a point A on the great circle, let A' denote its antipodal point. We'll assume that there are two antipodal points P and P' that have different temperatures (if not, then we're done trivially). Let t be the temperature function on the great circle, so $t(P) \neq t(P')$.

Define the temperature difference function d on the great circle by $d(A) = t(A) - t(A')$. Note that

$$d(P') = t(P') - t(P) = -(t(P) - t(P')) = -d(P) \neq 0,$$

so that $d(P)$ and $d(P')$ must have opposite signs. Moreover, d is a continuous function because t is continuous. So we may use the Intermediate Value Theorem, which implies there is some point Q between P and P' where $d(Q) = 0$. But if $d(Q) = 0$, then $t(Q) = t(Q')$; in other words Q and Q' are the antipodal points we're looking for.