

## Challenge of the Week

October 7–October 13, 2008

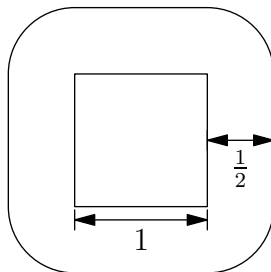
### Problem

120 unit squares are arbitrarily arranged in a  $20 \times 25$  rectangle (both position and orientation is arbitrary). Prove that it is always possible to place a circle of unit diameter inside the rectangle without intersecting any of the squares.

### Solution

The circle is required to lie inside the rectangle, so its center must be at least  $1/2$  a unit away from each edge. In other words, the center of the circle must lie in an inner  $19 \times 24$  rectangle having area 456.

The circle will intersect a particular unit square if its center lies within  $1/2$  a unit of the square, that is, when the center lies in the “forbidden” region shown below:



By breaking up the forbidden region into rectangles and quarter circles, we can calculate its area to be  $3 + \pi/4$ .

In the worst possible case, the 120 squares could be placed so that their forbidden regions would not overlap, so the center of the circle must avoid an area as large as  $120(3 + \pi/4) = 454.248$ . Since this area is less than that of the  $19 \times 24$  rectangle, there are some points where we can place the circle without it intersecting a square.