**Solution to the November 4, 2008 Challenge**

**Jack Lee (lee@math.washington.edu)**

**Answer:** The minimum possible sum of the numbers along a diagonal is 88. Here is a pattern that realizes it:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 2 | 23 | 24 | 25 | 26 | 27 | 28 |
| 64 | 3 | 22 | 21 | 20 | 31 | 30 | 29 |
| 63 | 4 | 5 | 6 | 19 | 32 | 33 | 34 |
| 62 | 61 | 60 | 7 | 18 | 17 | 16 | 35 |
| 57 | 58 | 59 | 8 | 9 | 10 | 15 | 36 |
| 56 | 55 | 54 | 53 | 52 | 11 | 14 | 37 |
| 47 | 48 | 49 | 50 | 51 | 12 | 13 | 38 |
| 46 | 45 | 44 | 43 | 42 | 41 | 40 | 39 |

**Proof:** Color each square black or white like those on a chessboard. We will prove that the numbers on the white diagonal sum to at least 88; the same argument applies by symmetry to the black diagonal.

The rules force the numbers in the white squares to be either all odd or all even. Thus the seven smallest numbers on the white diagonal must be greater than or equal to 1,3,5,7,9,11,13.

Now consider the largest number on the white diagonal. Imagine entering consecutive numbers into squares one at a time according to the given rule. As soon as the $8^{th}$ diagonal square is filled, the complement of the diagonal becomes disconnected, because it is not possible to move from a square in the upper-right half of the board to a square in the lower-left half (or vice versa) without traversing a square along the white diagonal. Thus, by the time the last diagonal square is filled, all the squares on one side of the white diagonal (by symmetry, we might as well say above it) must also have been filled. In particular, the 20 white squares on and above the white diagonal must be filled, which means that the last number entered into the diagonal must be at least as large as the $20^{th}$ odd number (39) or the $20^{th}$ even number (40). The above picture shows that 39 is possible, so 1+3+5+7+9+11+13+39 = 88 is the smallest possible sum.