

## Challenge of the Week

November 18–November 24, 2008

### Problem

A pencil, eraser, and notebook together cost one dollar. A notebook costs more than two pencils, three pencils cost more than four erasers, and three erasers cost more than a notebook.

How much does each item cost (assuming that each item costs an integral number of cents)?

### Solution

(From Justin Shih).

Letting  $p$ ,  $e$  and  $n$  denote the prices of the pencil, eraser, and notebook in cents, we have

$$n > 2p \tag{1}$$

$$3p > 4e \tag{2}$$

$$3e > n. \tag{3}$$

If we chain these inequalities together, then we find

$$p > (4/3)e \tag{4}$$

$$3e > n > 2p \quad (\text{so } p < (3/2)e) \tag{5}$$

$$n > 2p > (8/3)e.$$

Combining these, we get

$$5e = e + (4/3)e + (8/3)e < p + e + n = 100 < e + (3/2)e + 3e = (11/2)e$$

so that  $e < 20$  and  $e > 18$ , hence  $e = 19$ .

From (4) and (5), we get  $26 \leq p \leq 28$ .

- $p = 26$  yields  $n = 100 - e - p = 55$  satisfies all of the given conditions;
- $p = 27$  yields  $n = 54$  fails  $n > 2p$ ; and
- $p = 28$  yields  $n = 53$  fails  $n > 2p$ .

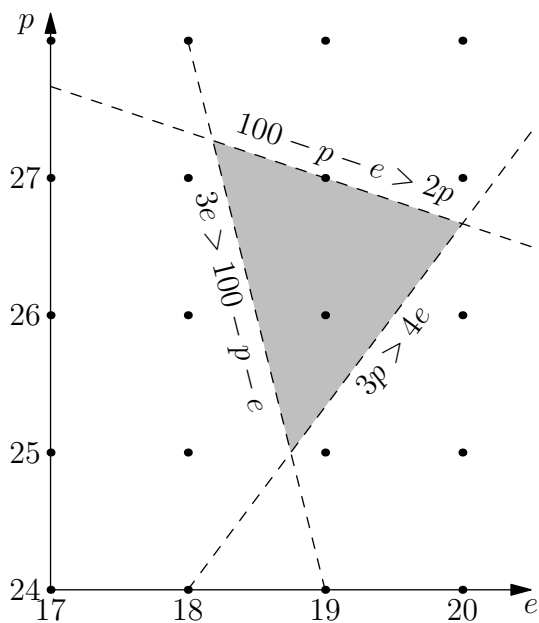
Thus we find the solution  $p = 26$ ,  $e = 19$ , and  $n = 55$ .



Gary Raymond pointed out that “nothing is surer than death or taxes.” In Seattle, the sales tax is \$0.09. If you buy the items separately, then the pencil would cost 24 cents + 2 cents tax, the notebook would cost 50 cents + 5 cents tax, and the eraser would cost 17 cents + 2 cents tax, totalling \$0.91 for the items + \$0.09 for the tax.



An alternate method is to substitute  $n = 100 - p - e$  into inequalities (1), (2), and (3) and graph them:



The only integer point satisfying all the inequalities is  $e = 19$ ,  $p = 26$ , giving  $n = 55$ .