Challenge of the Week

January 6–January 12, 2009

Problem

A positive integer is written on each face of a cube. Each corner is then assigned the product of the numbers on the three adjacent faces. The sum of the numbers assigned to the vertices is 1001. Find the sum of the numbers written on the faces of the cube.

Solution

Let the integers on the top, bottom, left, right, front, and back faces of the cube be t, b, l, r, f, k, respectively. Then the sum of the numbers on the vertices is:

$$tfl + tfr + tkl + tkr + bfl + bfr + bkl + bkr = (t + b)(f + k)(l + r) = 1001.$$

Note that 1001 has prime factorization $7 \cdot 11 \cdot 13$. Since each of t, b, l, r, f, k are positive, each of the factors $(t+b)$, $(f+k)$, and $(l+r)$ must be greater than one, so that $t+b$, $f+k$ and $l+r$ must be the numbers 7, 11, and 13 in some order.

So the sum of the integers on the faces is $t + b + l + r + f + k = 7 + 11 + 13 = 31$.