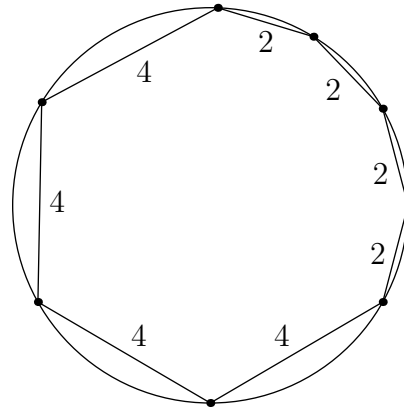


Challenge of the Week

January 13–January 19, 2008

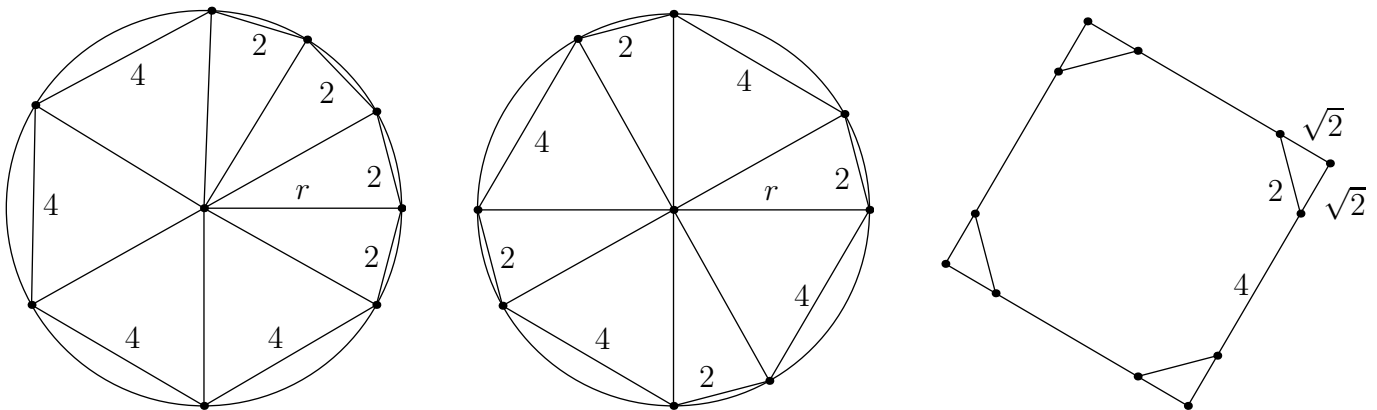
Problem

A convex octagon inscribed in a circle has side lengths 4, 4, 4, 4, 2, 2, 2, and 2. Find the area of the octagon.



Solution

Let r be the radius of the circle. The octagon is made up of four triangles sides 4, r , r and four triangles sides 2, r , r . The area is unchanged if we rearrange these triangles to form an octagon with sides alternately 2 and 4:



The new octagon is a square, side length $4 + 2\sqrt{2}$, with four corners lopped off. The parts lopped off are triangles with each sides 2, $\sqrt{2}$ and $\sqrt{2}$, which each have area 1.

Hence the area of the octagon is $(4 + 2\sqrt{2})^2 - 4 = 20 + 16\sqrt{2}$.