Challenge of the Week

January 27–February 2, 2008

Problem:

n points are marked on a circle. Two players take turns connecting them with line segments. Each new segment must be connected with the previous one, and it is not allowed to draw the same segment twice. The first player unable to make a segment loses.

Which player has a winning strategy? Describe the winning strategy.

Solution:

The first player can always win.

Notice that the maximal possible number of segments emitting from any vertex is $(n-1)$. Suppose on his first move the first player connects vertices A and B; therefore, the sum of the other segments emitting from A and B is $m = 2(n-2)$. No matter how the second player responds, the first player draws a segment back to A. The second player is forced to draw some segment emitting from A. The first player draws a segment back to B. The first player continues to alternate between returning to A and returning to B each move. Since m, the number of segments emitting from A and B is even, the first player takes the last segment to A (or B). After that, the second player has no move and loses.