Problem

This nifty problem was suggested by Tom Boothby. It arose while finding efficient algorithms to do arithmetic in finite fields for SAGE. A bit of background is necessary.

Define the four "inputs"

$A =$

0	1	1
0	1	1
0	1	1

$a =$

0	0	1
0	0	1
0	0	1

$B =$

0	0	0
1	1	1
1	1	1

$b =$

0	0	0
0	0	0
1	1	1

consisting of 3×3 arrays of 1's and 0's. We are allowed to combine these inputs together using the logical operations $\wedge$ (and), $\vee$ (or), $\oplus$ (exclusive or). Associating "true" with a 1, and "false" with a 0, the following table summarizes the operations.

r	s	$r \wedge s$	$r \vee s$	$r \oplus s$
0	0	0	0	0
0	1	0	1	1
1	0	0	1	1
1	1	1	1	0

We apply these operations to each of the cells in the 3×3 grids, so for example, we have

$A \wedge b =$

0	0	0
0	0	0
0	1	1

$A \vee b =$

0	1	1
0	1	1
1	1	1

$A \oplus b =$

0	1	1
0	1	1
1	0	0

Suppose we wish to make the outputs V and W from combinations of A, a, B and b.

$V =$

0	0	0
0	0	1
1	1	1

$W =$

0	0	1
0	0	1
0	1	1

We can make V and W in four steps as follows:

1. Let $C = a \wedge B$
2. Let $D = b \wedge A$
3. Let $V = C \vee b$
4. Let $W = D \vee a$

But if we're clever, we can do it with only three steps, like this:

1. Let $E = a \vee b$
2. Let $V = E \wedge B$
3. Let $W = E \wedge A$

Now we're ready for the puzzle: Starting from the four inputs A, a, B and b, make both the outputs

$X =$

0	0	1
0	1	0
1	0	0

$Y =$

0	1	1
1	1	0
1	0	1

The winner will be the one who accomplishes this using the fewest number of steps.