

Challenge of the Week

May 19–May 25, 2009

Problem

Each of ten girls around a circle chooses a number and tells it to the neighbor on each side. (Thus each person gives out one number and receives two numbers.) Each girl then announces the average of the two numbers she received. Remarkably, the announced numbers, in order around the circle, were 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

What was the number chosen by the girl who announced the number 6?

Solution

Suppose girl i chooses the number x_i . The sum of the numbers received by each girl is twice the average she announces. Thus we can form a set of ten equations:

$$\begin{array}{ll} x_{10} + x_2 = 2 & \text{girl 1 announces 1} \\ x_1 + x_3 = 4 & \text{girl 2 announces 2} \\ x_2 + x_4 = 6 & \text{girl 3 announces 3} \\ \vdots & \\ x_8 + x_{10} = 18 & \text{girl 9 announces 9} \\ x_9 + x_1 = 20 & \text{girl 10 announces 10} \end{array}$$

Solving these 10 equations is not difficult; we get

girl	1	2	3	4	5	6	7	8	9	10
number	6	-3	-2	9	10	1	2	13	14	5

In particular, girl 6 chose the number 1.

◇ ◇ ◇

Several people saw an elegant way to get the answer without solving everything by brute force, by working their way around the circle:

$$\begin{aligned} 2x_6 &= (x_6 + x_8) - (x_8 + x_{10}) + (x_{10} + x_2) - (x_2 + x_4) + (x_4 + x_6) \\ &= 14 - 18 + 2 - 6 + 10 \\ &= 2 \end{aligned}$$

So $x_6 = 1$.