

Challenge of the Week

May 26–June 1, 2008

Problem

Evaluate the following integral:

$$\int_3^9 \frac{\log(10-x)}{\log(10-x) + \log(x-2)} dx$$

Solution

Let I be the value of the integral in question. Making the substitution $u = x - 6$, we get

$$I = \int_{-3}^3 \frac{\log(4-u)}{\log(4-u) + \log(4+u)} du \quad (1)$$

From here, making the additional substitution $v = -u$ and interchanging limits, we get

$$I = \int_{-3}^3 \frac{\log(4+v)}{\log(4+v) + \log(4-v)} dv \quad (2)$$

Adding together (1) and (2) we get

$$2I = \int_{-3}^3 \frac{\log(4+x) + \log(4-x)}{\log(4+x) + \log(4-x)} dx = \int_{-3}^3 1 dx = 6$$

Thus $I = 3$.