Challenge Of the Week

March 2—March 8, 2010

**Problem:**

A square grid of side 4398046511104 has its left-top corner removed. Is it possible to fill this new grid with pieces of the form (rotations of the pieces are allowed) without overlapping such pieces?

**Solution:**

We show that it is possible to tile an $n \times n$ grid missing a corner if and only if $n$ is not a multiple of 3.

The picture below shows how to tile $n \times n$ squares for $n = 2, 4, 5,$ and 7 (shown in red), as well as how to tile $6 \times n$ rectangles for $n = 2, 4, 5, 6, 7$ (shown in blue and yellow).
We can use these tilings to make tilings for larger size grids. Write $n$ in the form $n = 6k + r$, where $2 \leq r \leq 7$.

- Suppose $n = 6k + 2$. To tile an $n \times n$ square missing a corner, put a red $2 \times 2$ piece in the top-left, $k$ blue $2 \times 6$ pieces on the left side, use $k$ more blue pieces on the top side, and fill the remaining bottom-right with $k^2$ yellow $6 \times 6$ squares.

- If $n = 6k + 3$ then $n$ is divisible by 3, so that $n^2$ is also divisible by 3. But then the grid missing a corner has $n^2 - 1$ squares, which is not divisible by 3, so a non-integral number of triominoes are needed. Such a tiling is impossible.

- If $n = 6k + 4$ then use the same approach as when $n = 6k + 2$, but use the $4 \times 4$ red square in the corner and $6 \times 4$ blue pieces for the sides.

- If $n = 6k + 5$ again use the same approach; use the $5 \times 5$ red square in the corner and $6 \times 5$ blue pieces for the sides.

- If $n = 6k + 6$ then $n$ is divisible by 3, so no tiling is possible.

- If $n = 6k + 7$ use the same approach; use the $7 \times 7$ red square in the corner and $6 \times 7$ blue pieces for the sides.

For this particular problem, we have a grid of size

$$n = 4398046511104 = 6 \cdot 733007751850 + 4,$$

so it is possible to tile using triominoes.