Final Review - Math 120

- Chapter 1, 2, 3, 4 - The Coordinate Plane, Lines and Circles
  - You should understand the idea of imposing a coordinate system and the use of the distance formula.
  - You should understand the various forms of the equations of a line, and be able to determine a line given either two points on the line, or the slope and a point on a line.
  - You should understand the equation of a circle, and be able to find the intersections of lines with lines, and lines with circles. You should be able to do this in the context of a model.
  - You should be able to create a linear model for the relationship between two quantities (e.g., population varying with time). If you have two such models, you should be able to answer various questions about the two quantities modeled (e.g., when are they equal? when is one twice as large as the other?)
  - Especially good problems include 2.13, 2.14, 3.4, 3.7, 4.8, 4.12, 4.14, 4.15.
- Chapters 5, 6 - Functions and Graphs
  - You should know a function is. You should know what the domain, range and graph of a function are, and, if asked be able to find the domain and range of a particular function. You should know what the *vertical line test* is all about.
  - You should be able to graph linear functions, and multipart functions whose parts are linear.
  - You should understand what a multipart function is. You should be able to describe relationships between two quantities with a multipart function, i.e., you should be able to model with them.
  - Especially good problems include 5.5, 5.10, 6.3, 6.4.
- Chapter 7 - Quadratic Functions
  - You should know that quadratic functions are those of the form $f(x) = ax^2 + bx + c$ and that these can always be put into vertex form $f(x) = a(x-h)^2 + k$. You should be able to find the vertex of a quadratic function.
  - You should be able to create quadratic models given three generic points, or the vertex and one other point.
  - You should be able to find the maximum or minimum value of a quantity determined by a quadratic function by considering the vertex.
  - I like problems 7.9-7.13 a lot.
- Chapter 8 - Composition
  - You should know what it means to compose two functions. You should understand what is meant by $f(g(x))$. You should know that $f(g(x))$ and $g(f(x))$ are generally different functions. You should be able to write simplified rules for compositions $f(g(x))$ and $g(f(x))$ given rules for $f(x)$ and $g(x)$.
  - I particularly like problems 8.3, 8.4 and 8.5.
- Chapter 9 - Three Construction Tools
  - You should understand horizontal and vertical **shifting**, and horizontal and vertical **scaling** (aka dilating)
  - You should understand how to derive the graph of $g(x) = af(bx + c) + d$ from the graph of $f(x)$ (see, e.g., problem 9.2)

  - I especially like problem 9.2, 9.3, and 9.7

- Chapter 10 - Arithmetic
  - This is a very short chapter. An important topic in this chapter is **step functions**, which are a nice example of multipart functions.
  - You should understand how to graph functions built up from the unit step function (see problem 10.8)
  - You should be able to combine multipart functions and come up with the rule for the new function.
  - I really like problems 10.4, 10.5 and 10.8.

- Chapter 11 - Inverse Functions
  - Another very short chapter.
  - You should understand what an **inverse function** is, what conditions a function must satisfy in order to have an inverse (do all functions have inverses? can you tell if a function has an inverse by looking at its graph?), and how to find the inverse of a given function
  - You should understand what a **one-to-one function** is, and what is special about the graph of a one-to-one function
  - I like problems 11.7 and 11.8.

- Chapter 12 - Rational Functions
  - A very important chapter. We spent two days in lecture on this instead of the usual one.
  - You should be able to find the **asymptotes** (horizontal and vertical) of a **rational function**, and be able to sketch the graph of a rational function like those in problem 12.1
  - You should be able to model with **linear-to-linear rational functions**. This comes down to finding a rational function of the form

    $$f(x) = \frac{ax+b}{x+c}$$

    whose graph

    1. passes through three given points
       or
    2. has a given asymptote and passes through two given points
       or
    3. has two given asymptotes and passes throuh one given point

    You will need to translate the language of the modeling problem.
    Pay particularly close attention to the words “linear-to-linear”.
    Note that a linear-to-linear function is not a **linear function**.
  - I especially like problems 12.1, 12.7, 12.8, 12.9, 12.11 and 12.12.

- Chapter 13 - Measuring an Angle
  - You should understand how to convert between **degrees** and **radians**
  - You should understand and be able to use the relationships between **radii**, **angle**, **arc length** and **area**
  - I like problems 13.8 and 13.9.

- Chapter 14 - Measuring Circular Motion

  - You should understand the various measures of **angular speed** (aka **angular velocity**), like rpm, radians per second, or degrees per hour
  - You should understand the relationship between **radius**, **angular speed** and **linear speed**
  - You should know how solve a belt-and-pulley problem (e.g., the bicycle example from lecture, example 14.4.1, problems 14.3, 14.9 and 14.11)
  - I like problems 14.5 and 14.7.

- Chapter 15 - The Circular Functions
  - This chapter introduces the **trigonometric functions**.
  - You should be able to solve problems using the idea of trigonometric functions as ratios of sides of right triangles (e.g., problems 15.4, 15.7, 15.8) and some algebra
  - You should understand the definitions of $\sin x$ and $\cos x$ using the **unit circle**; you should be able to determine certain simple properties of the functions $\sin x$ and $\cos x$ from this definition (e.g., the range, the domain, the graph, the values at certain value of x, like $x = 5\pi/2$)
  - You should be able to determine the location of an object moving circularly given information about its speed and starting location (e.g., problems 15.2, 15.5, 15.9, 15.14)

- Chapter 16 - Trigonometric Functions
  - This is a short chapter which adds some final touches to our knowledge of the functions $\sin x$ and $\cos x$ and related functions.
  - I like problems 16.3 and 16.4

- Chapters 17, 18 - Sinusoidal Functions
  - You should understand the notion of a **sinusoidal function** as a shifted/dilated version of the function $\sin x$.
  - You should understand the effect of the four parameters A, B, C and D on the graph of

$$f(x) = A \sin\left(\frac{2\pi}{B}(x - C)\right) + D.$$

  - You should be able to model with sinusoidal functions. In particular, you should be able to determine the parameters A, B, C, and D from a verbal description of a quantity that varies sinusoidally with time (see problems 17.2, 17.3, 17.4,17.6)
  - You should be able to solve equations of the form $f(x) = k$ where f is a sinusoidal function; if there are any solutions, there are infinitely many, and you should be able to find them. You should be able to do this in the context of a modeling problem (e.g., problems 18.2, 18.4, 18.6, 18.10, 18.11, 18.12)

- Chapters 19, 20, 21 - Exponential functions, modeling and logartithms
  - You should be able to recognize functions of the form $f(x) = A_0 b^x$ or, equivalently, $f(x) = A_0 e^{kx}$. You should be able to put exponential functions into these forms.
  - You should be able to create exponential models of quantities that change over time. Given two values of the quantity at two data points in time, you should be able to come up with an exponential model that fits the data. Given a single data point and information about the quantity's rate of growth (e.g., percentage annual increase, or doubling time), you should be able to come up with an exponential model that fits.
  - You should be able to solve equations involving exponential functions using the natural logarithm.
  - Relevant problems include 20.1,20.2, 21.9,21.11, 21.12, 21.15 and 21.16.