Writing Problem #3 - Math 125 Honors - Winter Quarter 2009

The **gamma function** is defined for real values of x by

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t}\, dt$$

1. Show that the integral above converges for all $x > 0$ and diverges for all $x \leq 0$.
2. Show that

   $$\Gamma(1) = 1.$$

3. Show that $\Gamma(x+1) = x\Gamma(x)$ for $x > 0$.
4. Conclude that, for $n \geq 2$ an integer,

   $$\Gamma(n) = (n-1)!.$$