Writing Problem #2

This problem is about investigating the function $f(x, y) = x^y$, and more particularly, the expression 0^0.

The expression 0^0 is undefined. One reason for this is that the function x^y is defined for real values of x and y by

$$x^y = e^{y \ln x}$$

and since $\ln 0$ is not defined, 0^0 isn't either.

However, if

$$\lim_{(x,y)\to(0,0)} x^y$$

existed, then perhaps the value of this limit would be a good choice for the value of 0^0.

For this problem, you should investigate this limit.

1. What is the limit of $f(x, y)$ as (x, y) approaches the origin along lines?
2. What is the limit of $f(x, y)$ as (x, y) approaches the origin along power curves, $y = x^p$?
3. Find other curves along which $f(x, y)$ approaches a limit different from the ones you found in (1) and (2).
4. What do you conclude about

   $$\lim_{(x,y)\to(0,0)} x^y?$$

   What is the surface $z = x^y$ like near the origin?