Review for the Final Exam - Math 124F - Autumn 2012

The primary topics for this exam are:

- **derivatives**
- **limits (including L'Hospital's rule)**
- **related rates**
- **linear approximation**
- **maxima and minima, inflection points and curve sketching**
- **optimization**

Here are some things to consider about each topic.

Derivatives

You should know how to differentiate everything. That is, you should be able to find $f'(x)$ given $f(x)$, and $f(x)$ could be any combination of algebraic expressions, trigonometric functions, logarithmic functions and exponential functions. Be actively aware of all rules of differentiation, e.g., product rule, quotient rule, chain rule.

You should understand and be able to use the technique of *logarithmic differentiation*.

Also, you should be able to find $\frac{dy}{dx}$ given an equation which relates y and x. Generally this means using *implicit differentiation*.

You should be able to determine $\frac{dy}{dx}$ of any point on a curve defined parametrically by, say, $x = f(t)$ and $y = g(t)$.

You should be able to find the tangent line to any curve given explicitly, implicitly or parametrically.

There are tons of problems in Stewart to practice. For starters, there are lots of problems in the sections on the chain rule, implicit differentiation and derivatives of logarithmic functions.

The first bunch of problems in the chapter 3 review can't be beat.

Limits and L'Hospital's Rule

You should be able to evaluate many sorts of limits.

It may be worth paying particular attention to practice non-L'Hospital techniques, as L'Hospital's rule can make us forget that sometimes other methods are needed. Think: what do I do if L'Hospital's rule does not help?

Some things to remember:

- L'Hospital's rule and the quotient rule are different things.
- You should check and **indicate** that the application of L'Hospital's rule is valid. Indicate this by writing $\frac{0}{0}$ or $\frac{\infty}{\infty}$ above the equal sign where the rule has been applied.

- You may need to apply L'Hospital's rule more than once to get a result. (Can you give an example of when this happens?)
- In some cases, even if L'Hospital's rule is applicable, another method may be simpler. Factoring and simplifying, for instance, may get you to the result more quickly than L'Hospital's rule.
- What happens if you apply L'Hospital's rule to this limit:

  $$\lim_{x\to\infty} \frac{x}{\sqrt{x^2+1}}?$$

  What method should you use instead?

Related Rates

Related rates problems come in a pretty wide variety; every problem in Stewart's section on related rates is good practice. Try to work a bunch of different ones, particularly from the later problems in the section.

There are also related rates problems in the chapter 3 review.

Be sure to practice a few problems of each of the various types: similar triangles, pythagorean theorem, volume/surface area os three-dimensional objects, rates of change of angles, etc.

Linear Approximation

The idea of linear approximation is that the tangent line can be used to approximate the function near the point of tangency.

You should be able to use linear approximations to approximate values of given functions. For instance, to approximate $\ln 53$, we can note that $e^4 = 54.598...$, so $\ln 53$ should be about 4, and we can get an even better approximation using the linear approximation

$$\ln 53 \approx \ln(e^4) + \frac{1}{e^4}(53 - e^4) = 4 + \frac{1}{e^4}(53 - e^4) = 3.97072886110291....$$

For such problems, the procedure to approximate $f(x)$ at some value b is to first find a value of a near b at which $f(a)$ is easy to calculate. Then, we can say

$$f(b) \approx f(a) + f'(a)(b - a).$$

In addition, you should be able to use linear approximations to approximate the location of a point on an implicitly defined curve. Here, the idea is to find a point on the curve near the point of interest, and then find the tangent line through that point; with that tangent line, the point of interest's location can then be approximated.

For example, to approximate the x-coordinate of the point on the curve

$$x^2 + y^3 = y/x + 1$$

with y-coordinate 0.9, we begin by noting that if $y = 1$, we have

$$x^2 + 1 = 1/x + 1$$

i.e., $x^3 = 1$, so $x = 1$. That is, $(1, 1)$ is on the curve. Finding the tangent line to the curve at that point, we have

$$y = -\frac{3}{2}(x - 1) + 1$$

and setting this equal to 0.9 and solving for x yields $x = 1.0\bar{6}$.

Maxima and Minima, Inflection Points, Curve Sketching and Optimization

You should be able to:

- Determine the maximum and minimum values of a function on a closed interval
- Classify local extrema using the first derivative test or second derivative test
- Determine the inflection points of a given function
- Sketch a curve, incorporating information about local extrema, increasing/decreasing regions, inflection points, and asymptotes
- Use all of these techniques to solve optimization problems

Keep in mind that the second derivative test can fail: sometimes you have to use the first derivative test. Can you give an example of when the second derivative test fails?

There are lots of good problems to practice all of the necessary techniques. A few good ones are found in section 4.5. Try problems 4.5 #5, 12, 13, 28, 39, 47.

To determine asymptotes, you may need L'Hospital's rule.

Optimization problems are a lot like related rates problems in that the setup is critical: the work you do before applying calculus is often the most the challenging part. Be sure to practice plenty of these kinds of problems. All of the problems in section 4.7 are good practice, but there are way too many to do all of them. Try a sampling. The problems that give you the function to be optimized will not give you practice with setting up such a function, so those problems are not as good for practice. Be sure to include some problems that involve trigonometry (e.g. 70-74), as these often have a different feel.

Keep in mind when solving optimization problems that it is never sufficient to just find a critical point. You must give calculations supporting your claim that this point yields a maximum, or minimum. Usually either the first-derivative test or second-derivative test is needed.