1. Give a useful denial of each statement. You may use symbols like $\forall$ and $\exists$, et cetera.

   (a) For every real number x, there exists a real number y such that $x < y$.

   (b) There exists a function $f : \mathbb{R} \to \mathbb{R}$ such that, for every real number x, $f(2x) = 4f(x)$.

   (c) If x is a positive real number, then $\ln x \geq 0$ or $\frac{1}{x} > 1$.

   (d) For all real x, $x \neq 0 \Rightarrow x^2 > 0$.

2. Let P be the statement: "Every prime number is odd." Which of the following are logically equivalent to P? (Check all that apply.)

   _____ n is odd implies n is prime.

   _____ n is odd if n is prime.

   _____ If n is odd, then n is prime.

   _____ If n is prime, then n is odd.

   _____ There exist numbers that are both prime and odd.

   _____ No even number is prime.

   _____ n is odd if and only if n is prime.

3. Show the following set equalities.

   (a) $(A \cup B) \cup (A \cap B) = A \cup B$

   (b) $(A \setminus B) \cup (B \setminus A) \cup (A \cap B) = A \cup B$

   (c) $(A \setminus B) \cap (A \cap B) = \emptyset$

   (d) $(A \cap B) \cap A = (A \cap B)$

4. Show the equivalence of the following statement pairs.

   (a) $(P \vee Q) \Rightarrow R$ and $\neg R \Rightarrow (\neg P \wedge \neg Q)$

   (b) $(P \Rightarrow Q) \Rightarrow R$ and $(P \wedge \neg Q) \vee R$

   (c) $P \Leftrightarrow Q$ and $(\neg P \vee Q) \wedge (\neg Q \vee P)$

5. Use quantifiers to write each of the following statements.

   (a) Every integer can be written as the sum of two squares.

   (b) There exists a real number r such that $rx = 0$ for every real number x.

   (c) For every real number x, we can find exactly one real number y such that $x + y = 0$.

6. Write the negation of each of the following. Is each statement true or false? For all statements, the universe is $\mathbb{Z}$.

   (a) $\exists x \exists y ((xy = 100) \wedge (x + y = 30))$

   (b) $\forall x \exists! y (x + y = 7)$

   (c) $\exists a \forall b (ab = b)$