Homework 3 - Math 300 D - Winter 2014 - Dr. Matthew Conroy
Relevant readings: Velleman, sections 2.1, 2.2, 2.3, 3.1, and 3.2.

1. Write useful negations of the following statements (in most cases, you should write the statement symbolically with quantifiers, negate the resulting expression, and then rewrite in english).
   (a) For every real number x, there exists a real number y such that $x + y = 0$.
   (b) There exists a real number z such that for all real numbers y, $zy = 0$.
   (c) For all pairs (x, y) of real numbers, there is a real number k such that $x^k + y^k = 2$.
   (d) Every integer n is prime or composite.
   (e) There exist integers p and q such that p and q are prime, and $p + q = 1480$.
2. Let A and B be sets. Prove that $\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B)$.
3. Prove that there exist sets A and B such that $\mathcal{P}(A \cup B) \neq \mathcal{P}(A) \cup \mathcal{P}(B)$.
4. Let a and b be negative real numbers. Prove that if $a < b$ then $a^2 > b^2$.
5. Let a, b and c be positive integers. Prove that if $a|b$ and $b|c$, then $a|c$.
6. One fact we use all the time when writing proofs is that, if $A \to B$ and $B \to C$, then $A \to C$. Prove this is valid by showing that

$$((A \to B) \wedge (B \to C)) \to (A \to C)$$

   is a tautology. Do this by using applicable laws to show that this is equivalent to a statement which we know is a tautology.
7. Now that we know the irrational numbers exist, we should prove a few facts about them.

   You can use the following useful facts in your proofs. You do not have to prove them.

   Fact 1: The sum of rational numbers x=a/b and y=c/d is (ad+bc)/(bd).

   Fact 2: If a is rational, then -a is rational; if a is irrational, then -a is irrational.

   Prove the following theorems:
   (a) The sum of two rational numbers is a rational number.
   (b) The sum of a rational number and an irrational number is an irrational number.
   (c) The product of an irrational number and a non-zero rational number is an irrational number.
   (d) The sum of two irrational numbers may be a rational number.