Here are some comments related to grading of Homework 3.

1. Here are some valid ways to introduce a rational number.

   - Let $x \in \mathbb{Q}$. Then there exist integers a and b, $b \neq 0$, such that $x = \frac{a}{b}$.
   - Let $x \in \mathbb{Q}$. Then $x = \frac{a}{b}$ for some integers a and b, $b \neq 0$.
   - Let $x = \frac{a}{b} \in \mathbb{Q}$, where $a, b \in \mathbb{Z}$, $b \neq 0$.

   If is not valid to say: "Let $a, b \in \mathbb{Z}, b \neq 0$. Let $x = \frac{a}{b}$." The reason this is not valid is that it is not immediately clear that x could take on all possible rational values. More immediately, what you are trying to do is introduce a rational number; the fact that it is expressible as a quotient of integers is secondary, and so it should come second.

2. People need to be putting in the axioms and elementary properties to justify steps in algebra. For instance, if I have shown that $a = bc$ and $b = cd$, then I may write

$$\begin{aligned} a &= (cd)c && \text{(Axiom 2 - substitution of equals)} \\ &= c(cd) && \text{(Axiom 3 - commutativity)} \\ &= (cc)(d) && \text{(Axiom 4 - associativity)} \\ &= c^2 d. \end{aligned}$$

   You need to put those Axioms (and as necessary, Elementary Properties) in to justify the steps in your proofs.

3. The word "since" is used without "then" or "so". For example, we say

   - Since A, B. (e.g., "Since I am hungry, I will eat lunch.")
   - A, so B. (e.g., "I am hungry, so I will eat lunch.")

4. When writing mathematics by hand, it is important that you are careful with your notation. Be sure to distinguish the Z representing the integers from just a general capital Z. Similarly for Q and R.

   When typeset these special sets look like $\mathbb{Z}, \mathbb{Q}$ and $\mathbb{R}$. The usual thing to do by hand is :

   (a) Double the diagonal part of the Z
   (b) Double the vertical part of the R
   (c) Add a vertical line to the Q, on the left side, a little ways inside the "O".

   All of these have been demonstrated in class.

5. You don't have to say A is a set if you are going to introduce the set explicitly.

   For example, instead of:

   "Let A be a set. Let $A = \{1, 2, 3\}$."

   we may just say

   "Let $A = \{1, 2, 3\}$."

The reason for this is that the statement $A = \{1, 2, 3\}$ tells us that A is a set; there is no need to state it separately.

6. Every proof you write from now on needs to be expressed in the form of a theorem-proof pair, as demonstrated in lecture and by your textbook.