

Math 300 D - Winter 2014 - Week 5 homework

Problems on relations and equivalence relations

Relevant reading: Velleman, 4.1, 4.2, 4.6 (Velleman says a ton more about relations in 4.3-4.5, if you are interested, but you won't need to know any of that stuff for this course)

1. How many equivalence relations are there on a set with three elements? List them.
2. For each of the following relations, determine whether it is reflexive, symmetric and transitive. Conclude whether or not the relation is an equivalence relation.

(a) Let $A = \mathbb{R}$. Define a relation R on A by

$$(x, y) \in R \Leftrightarrow x < y$$

(b) Let $A = \mathbb{R}$. Define a relation R on A by

$$(x, y) \in R \Leftrightarrow x \leq y$$

(c) Let $A = \mathbb{R} \times \mathbb{R}$.

Define a relation R on A by

$$((x_1, y_1), (x_2, y_2)) \in R \Leftrightarrow \text{the distance from } (x_1, y_1) \text{ to } (x_2, y_2) \text{ is a rational number.}$$

(d) Let $A = \mathbb{R} \times \mathbb{R}$.

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3. Let $a, b \in \mathbb{Z}$. Let $m \in \mathbb{Z}_{>0}$.

We say a is **congruent** to $b \pmod{m}$ iff $m \mid (a - b)$.

If a is congruent to $b \pmod{m}$, we write

$$a \equiv b \pmod{m}.$$

Prove that if $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$, then

$$(a + c) \equiv (b + d) \pmod{m}$$

and

$$ac \equiv bd \pmod{m}$$

4. Let $A = \mathbb{R}$.

Define a relation R on A by

$$(a, b) \in R \Leftrightarrow a - b \in \mathbb{Q}.$$

- (a) Show that R is an equivalence relation.
- (b) Give an example of one of the equivalence classes in A/R .
- (c) Prove that there are infinitely many equivalence classes in A/R .