

Math 300 D - Winter 2014 - Homework 6
Problems on modular arithmetic and functions
Relevant reading: Velleman, 4.6 and 5.1

1. Let $n \in \mathbb{Z}$. If $4 \mid n$, then $76n \equiv n \pmod{100}$.
2. Which digits can appear as the right-most digit of a squared integer? What about a cubed integer? State and prove a theorem.
3. Show that for all $m \in \mathbb{Z}, m \geq 0$,

$$49 \mid 5 \cdot 3^{4m+2} + 53 \cdot 2^{5m}.$$

4. Find the smallest positive integer k such that 26 divides $23^{78910} - k$.
5. Suppose $f : A \rightarrow C$ and $g : B \rightarrow C$. Prove that if $A \cap B = \emptyset$, then $f \cup g : (A \cup B) \rightarrow C$.
6. Suppose R is a relation on a set A . Is it possible that R is both a function and an equivalence relation? Complete and prove the statement “ R is a function and an equivalence relation iff ...”.
7. Let S and T be sets and $f : S \rightarrow T$. Define a relation R on S by

$$(a, b) \in R \Leftrightarrow f(a) = f(b).$$

Prove that R is an equivalence relation.