

Homework 8 - Math 300D - Winter 2014 - Dr. Matthew Conroy

1. Find the smallest  $k \in \mathbb{Z}$  such that  $n! > n^4$  for all  $n \geq k$ . Prove the result using induction.

2. Use induction to prove that

$$13 \mid 3^{6m+3} + 4^{6m+3}$$

for all  $m \in \mathbb{Z}_{\geq 0}$ .

3. Let  $n$  be a positive odd integer.

Use induction to prove that the sum of all positive odd integers less than or equal to  $n$  is

$$\left(\frac{n+1}{2}\right)^2.$$

4. Let  $A$  be a finite set. Prove that if  $f : A \rightarrow A$  is injective, then  $f$  is bijective.

5. Suppose  $A$  is an infinite set and  $B$  is a finite subset of  $A$ . Prove that  $A \setminus B$  is infinite.

6. Prove that, if  $A \sim B$ , then  $\mathcal{P}(A) \sim \mathcal{P}(B)$ .