

Here is a list of theorems and other facts that you can use without justification during the exam. This is only a partial list! Many minor results may be used without justification. This list is merely a reference of the more powerful and, perhaps, harder to remember ones.

- There are infinitely many primes.
- The transitivity of divisibility: if a divides b , and b divides c , then a divides c .
- If d divides a and d divides b , then d divides any linear combination of a and b .
- If d and n are positive integers, and d divides n , then $d \leq n$.
- If a prime p divides ab , then p divides a or p divides b .
- The Fundamental Theorem of Arithmetic: all positive integers can be written in a unique way as a product of primes.
- The Euclidean algorithm for finding the gcd of two integers
- Stark, Theorem 2.3: n is a common divisor of a and b iff n divides $\gcd(a, b)$.
- Stark, Theorem 2.6: If $(n, a) = 1$ and $n|ab$, then $n|b$.
- Stark, Theorem 2.13: The n -th root of a positive integer is rational iff it is an integer.
- The result from problem 6 in the week 2 homework (i.e., if $d|n$, then the prime factorization of d consists only of primes from the prime factorization of n , with exponents no greater than the corresponding exponents in the prime factorization of n).
- $\tau(n) = \prod_{p^\alpha || n} (\alpha + 1)$, $\sigma(n) = \prod_{p^\alpha || n} \frac{p^{\alpha+1} - 1}{p - 1}$, $\phi(n) = \prod_{p^\alpha || n} \left(1 - \frac{1}{p}\right)$
- For positive integers a and b , and any prime p ,

$$\text{ord}_p ab = \text{ord}_p a + \text{ord}_p b \text{ and } \text{ord}_p(a + b) \leq \min\{\text{ord}_p a, \text{ord}_p b\}$$

- The Chinese Remainder Theorem: Let m_1, m_2, \dots, m_k be pairwise relatively prime positive integers. Then the system $x \equiv a_1 \pmod{m_1}, x \equiv a_2 \pmod{m_2}, \dots, x \equiv a_k \pmod{m_k}$ has a unique solution modulo $m_1 m_2 \cdots m_k$.
- Euler's Theorem: Let n be a positive integer. Then $a^{\phi(n)} \equiv 1 \pmod{n}$ if $(a, n) = 1$.
- Frobenius Coin Theorem: Let $a, b > 0$ and $(a, b) = 1$. Then the equation $ax + by = m$ has no non-negative solutions (x, y) if $m = ab - a - b$ and does have solutions if $m > ab - a - b$.
- Primitive Pythagorean Triples: $x^2 + y^2 = z^2$ where $x, y, z \in \mathbb{Z}, x > 0, y > 0, z > 0, (x, y) = 1$ and $2 \mid x$ iff $x = 2ab, y = a^2 - b^2$ and $z = a^2 + b^2$ for some $a, b \in \mathbb{Z}, a > b > 0, (a, b) = 1$, and $a + b \equiv 1 \pmod{2}$.
- Generating functions: The generating function of a sequence $\{a_n\}$ is the function given by the power series $A(x) = \sum_{n=0}^{\infty} a_n x^n$.