Final exam practice problems
Math 301A, Spring 2014
Here are some problems worth doing as you prepare for the final exam. In addition, you should review all assigned homework problems. In particular, spend some time with any graded problems that you did not receive full points for, as these indicate places where something was lacking in your work.

1. Let a, b be positive integers. Prove that $(a, b) = 1$ iff there exists no prime p such that $p|a$ and $p|b$.

2. Prove that, if $(a, b) = 1$, then $a|b^2$ implies $a|b$.

3. How many primes are there of the form $m^2 - 25$?

4. Suppose p is a prime and for some integer a, $a^{12} \equiv 1 \pmod{p}$ and $a^{15} \equiv 1 \pmod{p}$. If r is the smallest positive integer such that $a^r \equiv 1 \pmod{p}$, then what can you say about the value of r?

5. Find all primitive pythagorean triples that include the number 34 (i.e., find all triangles with relatively prime integer sides, one of which is 34).

6. Prove that the product of four consecutive integers is congruent to 0 or 24 modulo 60.

7. Find all solutions to the system of equations

$$x \equiv 2 \pmod{3}$$
$$x \equiv 3 \pmod{4}$$
$$x \equiv 4 \pmod{5}$$

   then make up and solve similar systems. Post challenges on the discussion board!

8. Find all solutions to the equation

$$124x \equiv 12 \pmod{95}$$

   then make up and solve similar equations. Post challenges on the discussion board!

9. Notice that 322069 is divisible by 11, and $32 + 20 + 69 = 121$ and 121 is divisible by 11 (as is $1 + 21 = 22$). Use the idea this example suggests to come up with another test for divisibility by 11. Prove that your test works.

10. Find the last three digits of 432^{432}.

11. Prove that, if $(n, 100) = 1$, then $n^{20} \equiv 1 \pmod{100}$.

12. Find a solution to $2x + 3y = 5$ for which $|x| + |y| > 1000$.

13. Find the largest number that is not expressible as a sum of non-negative multiples of $5, 6$ and 7. Prove that the number you found is largest.

14. Write the generating function for each of the following sequences:

- $a_0 = 0, a_1 = 1, a_n = a_{n-1} + n$ for $n > 1$
- $a_0 = 0, a_1 = 1, a_n = 2a_{n-1} + 5$ for $n > 1$
- $a_0 = 0, a_1 = 1, a_n = a_{n-1} + 2a_{n-2}$ for $n > 1$

Can you use the generating function to find an expression for the general term of the sequence?

15. Find A and B such that the limit of the ratio of consecutive terms of the sequence defined by
$$a_n = Aa_{n-1} + Ba_{n-2}$$
is $5 + \sqrt{12}$.