

Homework 1 - Math 301 A - Spring 2014 - Dr. Matthew Conroy

1. Give an example of two even integers which can each be expressed as the sum of two primes in at least three ways (not counting ordering: e.g., $3+5$ and $5+3$ should be considered the same).
2. Prove that if $a \mid b$ and $b \mid a$, then $a = b$ or $a = -b$.
3. Prove that every integer > 1 has at least one prime divisor (do not use Harold's Theorem 1.5, which implicitly uses this result).
4. Start with a small set of small primes. Take their product and add one. Factor the result, and add the prime factors to your list. Repeat five times. What primes do you end up with? Do you think this process, continued indefinitely, would eventually include all primes?
5. Prove that if $n > 1$ is not prime, then it has a prime divisor $\leq \sqrt{n}$.
6. Let a be the number given by the first 5 digits of your student id number. Let b be the number given by the last 4 digits of your student id number. Find (a, b) and express it as a linear combination of a and b .
7. Show that if a, b and c are pairwise relatively prime, then $(a, b, c) = 1$. Show that the converse of this statement is false.
8. Show that if p is a prime and n is an integer, then $(p, n) = 1$ or $(p, n) = p$.