Homework 2 - Math 301 A - Spring 2014 - Dr. Matthew Conroy

1. Let n be a positive integer, and p a prime. Define $\mathrm{ord}_p n$ to be the value of k such that $p^k || n$. Prove the following properties of ord:

   (a) $\mathrm{ord}_p\, ab = \mathrm{ord}_p\, a + \mathrm{ord}_p\, b$

   (b) $\mathrm{ord}_p(a+b) \geq \min\{\mathrm{ord}_p\, a, \mathrm{ord}_p\, b\}$

2. (Note: you may want to prove, and use, number 6 first) Let a and b be positive integers. Suppose $\{p_1, p_2, \ldots p_k\}$ is a set of primes and that
$$a = p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}$$
and
$$b = p_1^{\beta_1} p_2^{\beta_2} \cdots p_k^{\alpha_k}.$$
Show that
$$\gcd(a,b) = p_1^{\min\{\alpha_1,\beta_1\}} p_2^{\min\{\alpha_2,\beta_2\}} \cdots p_k^{\min\{\alpha_k,\beta_k\}}.$$

3. Show that, for $n > 1$, $\sqrt[n]{n}$ is irrational.

4. How many zeros does $167890!$ end in, when written in decimal notation?

5. How many zeros does $66600!$ end in when it is written in base 8? Base 160? Base 224? (66600 is base 10 throughout). Explain the general method for finding the number of zeros at the end of $n!$ for each of these bases.

6. Suppose n has prime factorization $n = p_1^{a_1} p_2^{a_2} \cdots p_r^{a_r}$. Prove that a positive integer $d \mid n$ iff $d = p_1^{b_1} p_2^{b_2} \cdots p_r^{b_r}$ with $0 \leq b_i \leq a_i$ for $1 \leq i \leq r$.