Math 300 A - Spring 2014 - Midterm Review - Dr. Matthew Conroy

For the midterm, you should be comfortable (i.e., you should understand all terms and notation, be able to perform necessary calculations easily and accurately, and be capable of creating proofs of theorems involving these concepts, up to the level that you have seen in lecture, our textbook, and (especially in) the homework) with:

- prime numbers
- divisibility
- Euclidean algorithm and greatest common divisors
- Unique factorization
- irrational numbers
- the order of p in $n!$, "trailing" zeros in $n!$
- the arithmetic functions τ and σ

Problems for review

1. Prove that there are an infinite number of prime numbers.
2. Prove the *transitivity of divisibility*: If $a \mid b$ and $b \mid c$, then $a \mid c$.
3. (Stark, p.15) Suppose a and b are two consecutive odd primes. Prove that the prime factorization of $a + b$ involves at least three (not necessarily distinct primes). (For example, $5 + 7 = 12 = 2 \cdot 2 \cdot 3$.)
4. Let a, b, and c be integers. Prove that if $a \mid b$ and $a \mid c$, then $a \mid \frac{bc}{gcd(b,c)}$.
5. Let a and b be positive integers. Prove that if there exist integers r and s such that $d = ar + bs$, then $\gcd(a, b) | d$. Conclude that if there exist integers r and s such that $1 = ar + bs$, then $\gcd(a, b) = 1$.
6. Express the gcd of 2006 and 3540 as a linear combination of 2006 and 3540.
7. Let a, b and c be non-zero integers. Prove that $(a, b, c) = ((a, b), c)$.
8. How many zeros are there at the end of $2413!$? What if you write it in base 2 or base 12?
9. Prove that $29!$ has more than one million divisors.
10. Prove that the square root of $20!$ is irrational.
11. Prove that $(a^2, b^2) = (a, b)^2$.