MATH 300 C, Winter 2015
Midterm I Study Problems

1. Prove that, for all $x \in \mathbb{Z}$, if $x^2 - 1$ is divisible by 8, then x is odd.
2. Let a and b be integers. Prove that $x = a^2 + ab + b$ is odd iff a is odd or b is odd.
3. Let a and b be integers. Prove that $a(b + a + 1)$ is odd iff a and b are both odd.
4. Prove or give a counterexample for each of the following statements.
   (a) For all integers a and b, if $a|b$ and $b|a$, then $a = b$ or $a = -b$.
   (b) For all integers m and n, if $n + m$ is odd, then $n \neq m$.
5. Let A, B, and C be sets. Prove that $A \cap B = A \setminus (A \setminus B)$.
6. Let A, B and C be sets. Prove that $(A \cup B) \setminus (A \cup C) = B \setminus (A \cup C)$.
7. Let A, B and C be sets. Prove that $(A \setminus B) \setminus C = A \setminus (B \cup C)$.
8. Let A, B, and C be sets. Prove that $A \cup C \subseteq B \cup C$ iff $A \setminus C \subseteq B \setminus C$.
9. Write out the set (i.e., express the set by listing its elements) given by the expression
$$\mathcal{P}(\{1,2,3\}) \cap \mathcal{P}(\{2,3,4\}).$$
10. Let A and B be sets. Prove that
$$\mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B).$$
11. Let A and B be sets. Prove that $\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B)$.
12. Let A and B be sets. Prove that $A = B$ iff $\mathcal{P}(A) = \mathcal{P}(B)$.
13. Let A and B be sets. Prove that $A \cap B = \varnothing$ if and only if $\mathcal{P}(A) \cap \mathcal{P}(B) = \{\varnothing\}$. (Bonus: think about proving this with and without using # 11.)