Basic skills list for the 126 Final Exam

The following is a collection of some of the things you are expected to be able to do on the final exam. It is intended as a starting point, not as a comprehensive summary of the material. You are also expected to be able to combine these skills to solve more complex problems such as those that appeared in the assigned homework.

1. Vectors, basic

   You should be able to determine or find:

   (a) The magnitude of a vector
   (b) The **dot product** of two vectors
   (c) The **cross product** of two vectors
   (d) The angle between two vectors
   (e) Whether or not two vectors are parallel
   (f) Whether or not two vectors are perpendicular

2. Lines, planes, and points in 3D

   You should be able to determine or find:

   (a) The center and radius of a sphere given by its equation
   (b) The point of intersection of two lines
   (c) The line of intersection of two planes
   (d) The equation of a line passing through two given points
   (e) The equation of a plane passing through three given points
   (f) The equation of a plane passing through a point, parallel to a given plane
   (g) The equation of a plane containing a line and a given point
   (h) The angle between two intersecting planes
   (i) The angle between two intersecting lines
   (j) Whether or not a point is part of a given line, or a given plane
   (k) Whether or not a line is part of a given plane
   (l) Whether or not two planes intersect or are parallel
   (m) Whether or not two lines intersect or are parallel
   (n) Whether or not two sets of equations define the same, or different, lines or planes

3. Cylinders and Quadric Surfaces

   You should be able to:

   (a) identify a quadric surface from its equation
   (b) identify a cylinder given its equation (i.e., be able to tell that is is a cylinder)
   (c) identify a quadric surface from a sketch of one
   (d) describe the traces of a surface given its equation

4. Parametric and polar stuff

   You should be able to determine or find:

   (a) $\dfrac{dy}{dx}$ given $x = f(t)$ and $y = g(t)$
   (b) the tangent line to a curve defined parametrically
   (c) the arc length of (a piece of) a curve specified by $x = f(t)$, $y = g(t)$
   (d) the Cartesian equation of a curve defined using polar equations, and vice versa
   (e) the tangent line to a curve defined with a polar equation

   You should be able to sketch the graph of a curve defined by a simple polar equation.

5. Vector functions, space curves, and motion

   You should be able to:

   (a) Find the derivative $\vec{r}'(t)$ of a given vector function $\vec{r}(t)$
   (b) Find the arc length of a piece of a space curve defined by $\vec{r}(t)$
   (c) Find the curvature κ at a point on a space curve $\vec{r}(t)$ or on a planar curve $y = f(x)$ or $x = f(t), y = g(t)$.
   (d) Determine the unit tangent, principal unit normal, and binormal vector functions for a space curve $\vec{r}(t)$
   (e) Find the velocity and acceleration vector functions for a particle whose motion is specified by $\vec{r}(t)$

6. Functions of Several Variables

   You should be able to:

   (a) Describe and sketch the domain of a given two variable function
   (b) Sketch level curves of a given two variable function
   (c) Find the partial derivatives f_x, f_y, f_{xx}, f_{xy}, f_{yx}, and f_{yy} of a given two variable function $f(x, y)$
   (d) Find the equation of the tangent plane to a surface given by $z = f(x, y)$ at a point.
   (e) Give a linear approximation to a function $f(x, y)$ at a specified point.
   (f) Find the absolute maximum and minimum of a function $f(x, y)$ on a closed, bounded domain (e.g. a rectangle or disk in the plane).
   (g) Find and classify local extrema of a function $f(x, y)$.

7. Double Integrals

   You should be able to:

   (a) Evaluate a double integral of a function $f(x, y)$ over a region R in the xy-plane.
   (b) Use polar coordinates to evaluate double integrals.
   (c) Change the order of integration to evaluate a double integral.
   (d) Find the center of mass of a two-dimensional lamina with variable density.

8. Taylor Series

   You should be able to:

   (a) Find the Taylor polynomial of any specified degree of a given function (by repeated differentiation).

   (b) Determine a bound on the error between a function and one of its Taylor polynomials on a specified interval.

   (c) Determine an interval on which the error between a function and one of its Taylor polynomials is below some given bound.

   (d) Find the degree of the Taylor polynomial needed to achieve a specified degree of accuracy in approximating a given function on a given interval.

   (e) Determine the Taylor series of certain functions from the definition of a Taylor series.

   (f) Use substitution, differentiation, integration, series addition, and multiplication by powers of $x - b$ to create Taylor series from known series.

   (g) Determine the n-th derivative at b of a given function by first finding the function's Taylor series.

   Be sure to have these Taylor series on your notesheet, and know how to create other series from them:

$$\frac{1}{1-x} = \sum_{j=0}^{\infty} x^j \qquad \text{for } |x| < 1$$

$$e^x = \sum_{j=0}^{\infty} \frac{x^j}{j!} \qquad \text{for all } x$$

$$\sin x = \sum_{j=0}^{\infty} (-1)^j \frac{x^{2j+1}}{(2j+1)!} \qquad \text{for all } x$$

$$\cos x = \sum_{j=0}^{\infty} (-1)^j \frac{x^{2j}}{(2j)!} \qquad \text{for all } x$$