

## Writing Assignment - Math 381 - Dr. Matthew Conroy

Consider the following game played with a single die. Suppose the die is six-sided, and fair (in that each side is equally likely), but the 5 has been replaced by a 3. So the faces 1, 2, 4, and 6 each occur with probability  $1/6$  and the face 3 appears with probability  $2/6$ . There is no 5 face.

A player starts with a score of zero and throws the die repeatedly, until either winning or losing. They add the value of the face that appears on each throw of the die to their score.

If the score is ever a prime number less than 18 (2, 3, 5, 7, ...), the player loses.

If the score reaches a value of 18 or greater, the player wins.

Model this game using a Markov chain. Set up a transition matrix with two absorbing states (one for winning, one for losing). Use the transition matrix to determine the probability of the player winning the game. Be sure to thoroughly support all of your claims.

You need to give an exact value of the probability, not a numerical approximation. You either need to use software capable of this directly (I recommend Sage or PARI/GP), or figure out how to do the calculation entirely with integers.

Write up your solution from the point of view that this game and the calculation of the winning probability was **entirely your idea**. Imagine that **you** came up with the problem, so you have to explain everything to an unknown reader. **Do not quote the text on this page!** Put everything in your words.