Here is an exponential modeling example.

In 1980, City X had a population of 1.2 million. City X's population increases by 0.8% every year.

In 1990, City Y had twice as many people as City X, but only 85% more people than City X in 2005.

If City Y's population is growing exponentially, when will the populations be equal?

We begin by finding exponential models for each city. We will express these in units of millions.

Let $t = 0$ represent 1980, so that throughout t represents years after 1980.

Let P_X be the population of City X. Then we seek A_0 and b, constants, such that

$$P_X = A_0 b^t.$$

Since $P_X = 1.2$ when $t = 0$, we have $A_0 = 1.2$.

Then since the population increases by 0.8% per year, we know that $b = 1.008$.

In general, a constant increase of $r\%$ per year translates to a b value of $1 + \frac{b}{100}$.

Thus

$$P_X = 1.2(1.008)^t.$$

(Since $1.008 = e^{ln 1.008} = e^{0.0079681696}$, we may write

$$P_X = 1.2\left(e^{0.0079681696}\right)^t = 1.2e^{0.0079681696t}$$

which is of the form $A_0 e^{st}$, another popular form for exponential functions.)

For city Y, we first calculate the population of City X in 1990. We have

$$P_X = 1.2(1.008)^{10} = 1.2995.$$

In 1990, City Y's population was twice this, i.e., $P_Y = 2.5991$.

In 2005, City X had a population of $1.2(1.008)^{25} = 1.4645$. City Y was 85% more than this, so

$$P_Y = 1.85(1.4645) = 2.7094.$$

This gives us two data points for City Y. Assuming $P_Y = B_0 c^t$, we seek B_0 and c, from the equations

$$2.5991 = B_0 c^{10}$$

$$2.7094 = B_0 c^{25}$$

Dividing the first into the second, we find

$$\frac{2.7094}{2.5991} = 1.04244 = c^{15}$$

so $c = 1.04244^{1/15} = 1.0027746$.

Plugging this in to $2.5991 = B_0 c^{10}$ and solving for B_0, we find $B_0 = 2.5280$.

Thus $P_Y = 2.5280(1.0027746)^t$.

(Since $1.0027746 = e^{\ln 1.0027746} = e^{0.0027707}$, we may also write

$$P_Y = 2.5280\left(e^{0.0027707}\right)^t = 2.5280e^{0.0027707t}$$

which has the form $A_0 e^{st}$, another popular form for exponential functions.)

Setting $P_X = P_Y$, and solving t, we have

$$1.2(1.008)^t = 2.5280(1.0027746)^t$$

$$\ln 1.2 + t\ln 1.008 = \ln 2.5280 + t\ln 1.0027746$$
$$t = \frac{\ln 2.5280 - \ln 1.2}{\ln 1.008 - \ln 1.0027746} = 143.363.$$

Thus the populations will be equal 143.363 years after 1980.