Summary for the Final Exam - Math 120 - Winter 2011

Here is a summary of the topics for the final exam.
The core of your studying should be the assigned homework problems: make sure you really understand those well before moving on to other things (like the old midterms on the test archive). In the test archive, there are many old final exams for you to study with. The only topic you'll find in the old finals that you need not concern yourself with is the summing of multi-part functions. Other than that, every topic on old finals is fair game for your upcoming final exam.

- Chapter 1 - Warm Up
  - One of the most important ideas of this chapter is that of *multiplying by one* as a means of unit conversion. This idea makes all unit conversions have a common method, and helps one's notekeeping.
- Chapter 2 - Imposing Coordinates
  - This chapter introduced the use of the *coordinate system* and the *distance formula*.
  - A classic problem from this chapter is one in which two objects are moving and we need to describe the distance between them, like problems 2.3, and 2.10.
- Chapter 3 - Three Simple Curves
  - This chapter introduces circles and horizontal and vertical lines. You should be sure you are comfortable finding the equation of a circle from a variety of descriptions.
  - You should be able to find the intersection of a circle with a vertical or horizontal line.
- Chapter 4 - Linear Modeling
  - In this chapter, we get the general line definition. Be sure you are able to find the intersection of a given circle with a general line.
  - We also have the idea of perpendicular lines, and the method for finding the shortest distance between a line and a point not on that line. We also considered tangent lines to circles.
  - Uniform linear motion is introduced. See problems 4.14 and 4.15.
  - Especially good problems are 4.7, 4.9, 4.11, 4.12.
- Chapter 5 - Functions and Graphs
  - Here the *function* is introduced.
  - Every function has a domain, range and graph. Be sure to know what each is, and how to determine it for a given function. As we said, finding the range and graph can be hard; rest assured, if asked to find the range or graph of a given function, it will be doable.
  - You should be comfortable with *multipart* functions (what are they, how to evaluate one, how to solve equations involving them, etc.) What's an example of a multipart function?
  - I like problem 5.7 particularly.

- Chapter 6 - Graphical Analysis
  - Chapter 6 talks about a variety of function-related topics.
  - You should understand how to graph a multipart function, where each part is linear.
  - Especially good problems are 6.5, 6.6, 6.8, and 6.9.
- Chapter 7 - Quadratic Functions
  - You should know that quadratic functions are those of the form $f(x) = ax^2 + bx + c$ and that these can always be put into vertex form $f(x) = a(x - h)^2 + k$. You should be able to find the vertex of a quadratic function.
  - You should be able to create quadratic models given three generic points, or the vertex and one other point.
  - You should be able to find the maximum or minimum value of a quantity determined by a quadratic function by considering the vertex.
  - I like problems 7.7-7.15 a lot.
- Chapter 8 - Composition
  - You should know what it means to compose two functions. You should understand what is meant by $f(g(x))$. You should know that $f(g(x))$ and $g(f(x))$ are generally different functions. You should be able to write simplified rules for compositions $f(g(x))$ and $g(f(x))$ given rules for $f(x)$ and $g(x)$.
  - I particularly like problems 8.2, 8.3 and 8.4.
- Chapter 9 - Inverse Functions
  - You should understand what an **inverse function** is, what conditions a function must satisfy in order to have an inverse (do all functions have inverses? can you tell if a function has an inverse by looking at its graph?), and how to find the inverse of a given function
  - You should understand what a **one-to-one function** is, and what is special about the graph of a one-to-one function
  - I like problems 9.2, 9.5, and 9.7.
- Chapters 10, 11, 12 - Exponential functions, modeling and logartithms
  - You should be able to recognize functions of the form $f(x) = A_0 b^x$ or, equivalently, $f(x) = A_0 e^{kx}$. You should be able to put exponential functions into these forms.
  - You should be able to create exponential models of quantities that change over time. Given two values of the quantity at two data points in time, you should be able to come up with an exponential model that fits the data. Given a single data point and information about the quantity's rate of growth (e.g., percentage annual increase, or doubling time), you should be able to come up with an exponential model that fits.
  - You should be able to solve equations involving exponential functions using the natural logarithm.
  - Relevant problems include 11.1, 11.2, 12.7, 12.9, 12.10, 12.12, 12.13.

- Chapter 13 - Three Construction Tools
  - You should understand horizontal and vertical **shifting**, and horizontal and vertical **scaling** (aka dilating)
  - You should understand how to derive the graph of $g(x) = af(bx+c)+d$ from the graph of $f(x)$ (see, e.g., problem 13.2)
  - I especially like problem 13.2, 13.3, and 13.5.
- Chapter 14 - Rational Functions
  - You should be able to find the **asymptotes** (horizontal and vertical) of a **linear-to-linear rational function**, and be able to sketch the graph of a rational function like those in problem 14.1
  - You should be able to model with **linear-to-linear rational functions**. This comes down to finding a rational function of the form

    $$f(x) = \frac{ax+b}{x+c}$$

    whose graph
    1. passes through three given points
       or
    2. has a given asymptote and passes through two given points
       or
    3. has two given asymptotes and passes throuh one given point

    You will need to translate the language of the modeling problem.
    Pay particularly close attention to the words "linear-to-linear".
    Note that a linear-to-linear function is not a **linear function**.
  - I especially like problems 14.1, 14.3, 14.5, 14.6, 14.7, and 14.9.
- Chapter 15 - Measuring an Angle
  - You should understand how to convert between **degrees** and **radians**
  - You should understand and be able to use the relationships between **radii**, **angle**, **arc length** and **area**
  - I like problems 15.8 and 15.9.
- Chapter 16 - Measuring Circular Motion
  - You should understand the various measures of **angular speed** (aka **angular velocity**), like rpm, radians per second, or degrees per hour
  - You should understand the relationship between **radius**, **angular speed** and **linear speed**
  - You should know how solve a belt-and-pulley problem (e.g., the bicycle example from lecture, example 16.4.1, problems 16.2, 16.7 and 16.8)
  - I like problems 16.4 and 16.5

- Chapter 17 - The Circular Functions
  - This chapter introduces the **trigonometric functions**.
  - You should be able to solve problems using the idea of trigonometric functions as ratios of sides of right triangles (e.g., problems 17.2, 17.3, 17.7, and 17.8) and some algebra
  - You should understand the definitions of $\sin x$ and $\cos x$ using the **unit circle**; you should be able to determine certain simple properties of the functions $\sin x$ and $\cos x$ from this definition (e.g., the range, the domain, the graph, the values at certain value of x, like $x = 5\pi/2$)
  - You should be able to determine the location of an object moving circularly given information about its speed and starting location (e.g., problems 17.1, 17.4, 17.5, 17.6, 17.9, 17.10, 17.12, 19.6, 20.8)
- Chapter 18 - Trigonometric Functions
  - This is a short chapter which adds some final touches to our knowledge of the functions $\sin x$ and $\cos x$ and related functions.
  - You should be thoroughly familiar with the graphs of $y = \sin x$ and $y = \cos x$
  - I like problems 18.1, 18.2, 18.3, and 18.6.
- Chapters 19, 20 - Sinusoidal Functions
  - You should understand the notion of a **sinusoidal function** as a shifted/dilated version of the function $\sin x$.
  - You should understand the effect of the four parameters A, B, C and D on the graph of

    $$f(x) = A \sin\left(\frac{2\pi}{B}(x - C)\right) + D.$$

  - You should be able to model with sinusoidal functions. In particular, you should be able to determine the parameters A, B, C, and D from a verbal description of a quantity that varies sinusoidally with time (e.g., problems 19.2, 19.3, 19.4)
  - You should be able to solve equations of the form $f(x) = k$ where f is a sinusoidal function; if there are any solutions, there are infinitely many, and you should be able to find them. You should be able to do this in the context of a modeling problem (e.g., problems 20.3, 20.4, 20.5, 20.6, 20.7)