

Math 124 K - Autumn 2008
Mid-Term Exam Number One
October 21, 2008

Name: _____

Student ID number: _____

Section: _____

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|-------|----|--|
| 1 | 20 | |
| 2 | 10 | |
| 3 | 10 | |
| 4 | 10 | |
| 5 | 10 | |
| 6 | 10 | |
| 7 | 10 | |
| Total | 80 | |

- Complete all questions.
- You may use a scientific calculator during this examination; graphing calculators and other electronic devices are not allowed and should be turned off for the duration of the exam.
- If you use trial-and-error, a guess-and-check method, or numerical approximation when an exact method is available, you will not receive full credit.
- You may use one double-sided, hand-written, 8.5 by 11 inch page of notes.
- Show all work for full credit.
- You have 80 minutes to complete the exam.

1. Determine the values of the following limits or state that the limit does not exist. If it is correct to say that the limit equals ∞ or $-\infty$, then you should do so.

(a)
$$\lim_{x \rightarrow 0^+} \frac{1+x}{e^{x^2-x} - 1}$$

(b)
$$\lim_{x \rightarrow 3} \frac{\sqrt{x^2 - 9}}{\sqrt{2x - 6}}$$

(c)
$$\lim_{x \rightarrow 0^-} \frac{|x - |x||}{|2x - |x||}$$

(d)
$$\lim_{x \rightarrow \infty} \left(\frac{2x + \sin(x)}{x - 3 \sin(x)} + \frac{\sin(x^2 + x) - \sin(x^2 - x)}{x + 1} \right)$$

2. Find a point on the curve $y = x^3$ for which the tangent line at that point passes through the point $(0, 1)$.

3. Find the x -coordinates of all points on the curve $y = (x + 2)(x^2 - 8x + 1)$ at which the tangent line is horizontal.

4. Let f be defined by the expression $f(x) = x|x|$. If f is differentiable at $x = 0$, find $f'(0)$. If f is not differentiable at $x = 0$, explain why. In either case, be sure to show all your work.
5. Show that if f is an odd function, then the function g defined by $g(x) = |f(x)|$ is an even function.

6. The height, in meters, of a rocket above the ground t seconds after launch was given by the function

$$h(t) = 400t - 9.8t^2$$

- (a) Determine the time t at which the rocket's instantaneous velocity was 100 meters per second.

- (b) Find an interval of time starting at $t = 5$ during which the average velocity of the rocket was 100 meters per second. Give the length of this interval.

7. Determine the value of a and the value of b such that the following function, f , is continuous at $x = 0$.

$$f(x) = \begin{cases} \frac{\sqrt{ax+b}-5}{x} & \text{if } x < 0, \\ 1 & \text{if } x \geq 0. \end{cases}$$