

Math 124 K - Autumn 2009
Mid-Term Exam Number One
October 27, 2009

Name: _____

Student ID number: _____

Section: _____

1	20	
2	10	
3	10	
4	10	
5	10	
6	10	
Total	70	

- Complete all questions.
- You may use a scientific calculator during this examination; graphing calculators and other electronic devices are not allowed and should be turned off for the duration of the exam.
- If you use trial-and-error, a guess-and-check method, or numerical approximation when an exact method is available, you will not receive full credit.
- You may use one double-sided, hand-written, 8.5 by 11 inch page of notes.
- Show all work for full credit.
- You have 80 minutes to complete the exam.

1. Determine the values of the following limits or state that the limit does not exist. If it is correct to say that the limit equals ∞ or $-\infty$, then you should do so.

(a) $\lim_{x \rightarrow -2} \frac{x^2 - 4}{x + 2}$

(b) $\lim_{x \rightarrow 0^+} \frac{|2x - 4| + 4x - 4}{x + |x|}$

(c) $\lim_{x \rightarrow 0} \frac{\sqrt{x^2 + 3} - \sqrt{x + 3}}{x^2 - x}$

(d) $\lim_{x \rightarrow 0} \left(\frac{1}{x} - \frac{1}{x^2 + x} \right)$

2. Find the derivative of each of the following functions. You do not need to do any simplification.

(a) $f(x) = \frac{x^{12} - x + 6}{2x^5 + x \sin x}$

(b) $g(x) = x(\sec x)e^x$

3. Give the equations of two lines which are both tangent to the curve $y = x^2 - 2x$ and are perpendicular to each other.

4. Show that the function

$$f(x) = \begin{cases} x \sin\left(\frac{1}{x^3}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

is continuous at $x = 0$.

5. Prove, via an "epsilon-delta" argument, that

$$\lim_{x \rightarrow 2} (3x + 4) = 10.$$

6. Let $a > 0$. Determine the area cut off from the first quadrant by a line which is tangent to the curve $y = \frac{1}{x}$ at the point $(a, 1/a)$ (i.e., the area of the triangle determined by the tangent line, the x -axis and the y -axis).