Math 124 K - Autumn 2007
Mid-Term Exam Number Two
November 20, 2007
Answers

1. (a) $$f'(x) = 3x^2\cos(2x+e^x) - x^3\left(\sin(2x+e^x)\right)(2+e^x)$$

   (b) $$g'(x) = \frac{(1+3x^2)(x-\sin x) - (x+x^3)(1-\cos x)}{(x-\sin x)^2}$$

2. (a) $$\frac{dy}{dx} = e^{(x^4-1)\ln(x^3+1)}\left(4x^3\ln(x^3+1) + \frac{x^4-1}{x^3+1}3x^2\right)$$

   (b) $$\frac{dy}{dx} = \frac{\sin x - \frac{1}{y}}{\cos y - \frac{x}{y^2}}$$

3. There are four points: $\left(\frac{1}{2}, \frac{2}{\sqrt{3}}\right)$, $\left(\frac{1}{2}, -\frac{2}{\sqrt{3}}\right)$, $\left(-\frac{1}{2}, \frac{\sqrt{2}}{\sqrt{3}}\right)$, and $\left(-\frac{1}{2}, -\frac{\sqrt{2}}{\sqrt{3}}\right)$.

4. With $f(x) = \ln x + x^2$, the linearization $L(x)$ at a=0.75 is

   $$L(x) = f(0.75) + f'(0.75)(x - 0.75)$$

   Setting this equal to zero, and solving for x yields

   $$x = 0.75 - \frac{f(0.75)}{f'(0.75)} = 0.65300543.$$

   This is very close to the actual root, $0.65291864041920471553508\ldots$.

5. There are two points: $\left(\frac{1}{\sqrt{2}}, \sqrt[3]{1 - \frac{1}{2\sqrt{2}}}\right)$ and $\left(-\frac{1}{\sqrt{2}}, \sqrt[3]{1 + \frac{1}{2\sqrt{2}}}\right)$

6. The area of the sector is

   $$A = \frac{1}{2}r^2\theta$$

   and the rate of change of the area at the instant described is 96 cm^2/hr.

7. There are four points that need to be checked: the endpoints, and two critical points: $x = 1$ and $x = e^{2/3}$. The minimum occurs at $x = 1$ and the maximum occurs at $x = \frac{1}{2}$.